Large-Scale Robust Optimization in Challenging Scheduling Problems

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Outline

- Scheduling under Uncertainty
- Short-Term Scheduling Formulation
- Robust Optimization
  - Bounded Uncertainty
  - Uncertainty with Known Distribution
- Examples
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  - Case Study 2: Large scale Polymer Compounding Plant
- Conclusions

Scheduling Under Uncertainty

- Production scheduling has received increasing attention from both academia and industries in the past decade (Floudas & Lin, 2004, 2005).
- Uncertainty exists widely in process model parameters and environmental data: processing time, demand, market price, etc.
- Existing techniques:
  - Stochastic methods
  - Robust optimization methods
  - Probabilistic methods
  - Fuzzy set programming methods
  - Reactive scheduling

Scheduling Under Uncertainty

- Stochastic methods:
  - Bassett, Pekny, Reklaitis (1997) – I&ECR
  - Vin and Ierapetritou (2001) – I&ECR
  - Balasubramanian and Grossmann (2002) – I&ECR
  - Balasubramanian and Grossmann (2004) – C&ChE
  - Jia and Ierapetritou (2004) – I&ECR
  - Bonfill, Bagajewicz, Espuna, Puigjaner (2004) – I&ECR
  - Bonfill, Espuna, Puigjaner (2005) – I&ECR
  - Ostrovsky, Datskov, Achenie, Volin (2004) – AIChE J.
- Robust optimization methods:
  - Ben-Tal and Nemirovski (2000) – Math. Prog.
  - Lin, Janak, Floudas (2004) – I&ECR
  - Janak, Lin, Floudas (2005) – I&ECR
  - Bertsimas and Sim (2003) – Math. Prog. Ser. B
  - Bertsimas and Sim (2004) – Oper. Res.

Scheduling Under Uncertainty

- Probabilistic or chance constraints:
  - Orcun, Altinel, Hortacsu (1996) – C&ChE
  - Petkov and Maranas (1997) – I&ECR
- Fuzzy set programming methods:
  - Wang (2004) – Eur. J. Oper. Res.
  - Balasubramanian and Grossmann (2003) – C&ChE
- Reactive scheduling:
  - Cott and Macchietto (1989) – C&ChE
  - Kanakamedala, Reklaitis, Venkatasubramanian (1994) – I&ECR
  - Huercio, Espuna, Puigjaner (1995) – C&ChE
  - Sanmarti, Huercio, Espuna, Puigjaner (1996) – C&ChE
  - Rodrigues, Gimeno, Passos, Campos (1996) – C&ChE
  - Honkomp, Mockus, Reklaitis (1999) – C&ChE
  - Vin and Ierapetritou (2000) – I&ECR
  - Roslof, Harjunkoski, Bjorkqvist, Karlsson, Westerlund (2001) – C&ChE

Uncertainty

- Sources:
  - Process model parameters: processing time/rate
  - Environmental data: market demands, due dates, prices
- Types:
  - Discrete or continuous distribution
  - Characteristics: bounded, symmetric, etc.
  - Known distribution: normal, uniform, discrete, etc.

Short-Term Scheduling Model

Problem Statement

- Given
  - Production recipe in terms of task sequences
  - Pieces of equipment and their ranges of capacities
  - Intermediate storage capacity
  - Production requirement
  - Time horizon under consideration
  - Uncertain parameters and level/form of uncertainty
- Determine
  - Optimal sequence of tasks taking place in each unit
  - Amount of material processed at each time in each unit
  - Processing time of each task in each unit
- so as to optimize a performance criterion,
  - Maximization of production, minimization of makespan, etc.
  - Subject to the production schedule remaining feasible for all instances of the uncertain parameters

Novel Continuous-time Model: Concepts

- Continuous-time representation
  - event points: time instances when tasks begin
  - same number of event points in all units
  - different locations of event points for different units (unit specific continuous-time representation)
- Decouple task events from unit events
  - reduce number of binary variables
- Avoid nonlinearities
- Avoid use of time slots
- Reduce the combinatorial problem

Continuous-Time Formulation for Short-Term Scheduling

Ierapetritou and Floudas (1998a,b); Janak and Floudas (2004); Floudas and Lin (2004), (2005); Lin and Floudas (2001)

- Continuous-time representation
- Mixed Integer Linear Programming (MILP) Problem

Max/Min Profit/Make-span

s.t. Allocation constraints

Capacity constraints

Material balance constraints

Storage constraints

Demand constraints

Duration constraints

Sequence constraints

Time horizon constraints

Short-Term Scheduling Model

Variables:

$wv(i,j,n)$ binary, assign the beginning of task (i) in unit (j) at event point (n);

$yv(j,n)$ binary, assign the utilization of unit (j) at event point (n);

$B(i,j,n)$ amount of material undertaking task (i) in unit (j) at event point (n);

$STI(s)$ initial amount of state (s);

$ST(s,n)$ amount of state (s) at event point (n);

$STF(s)$ amount of state (s) at the end of the horizon;

$D(s,n)$ amount of state (s) delivered at event point (n);

$SL(s,n)$ slack variable for the amount of state (s) not meeting the demand at event point (n);

$T^s(i,j,n)$ time that task (i) starts in unit (j) at event point (n);

$T^f(i,j,n)$ time that task (i) finishes in unit (j) while it starts at event point (n).

Short-Term Scheduling Model

Allocation Constraints

$$\sum_{i \in I_j} wv(i,j,n) = yv(j,n), \forall j \in J, n \in N$$

Material Balance Constraints

$$ST(s,n_{1st}) = STI(s) + \sum_{i \in I_s} \rho_{si}^c \sum_{j \in J_i} B(i,j,n_{1st}), \forall s \in S$$

$$ST(s,n) = ST(s,n-1) - D(s,n) + \sum_{i \in I_s} \rho_{si}^p \sum_{j \in J_i} B(i,j,n-1)$$

$$+ \sum_{i \in I_s} \rho_{si}^c \sum_{j \in J_i} B(i,j,n), \forall s \in S, n \in N$$

$$STF(s) = ST(s,n_{last}) + \sum_{i \in I_s} \rho_{si}^p \sum_{j \in J_i} B(i,j,n_{last}), \forall s \in S$$

where $\rho^c_{si} \leq 0$, $\rho^p_{si} \geq 0$ represent the proportion of state (*s*) consumed by or produced from task (*i*), respectively.

Short-Term Scheduling Model

Capacity Constraints

$$\boxed{V_{ij}^{\min} \cdot wv(i,j,n) \le B(i,j,n) \le V_{ij}^{\max} \cdot wv(i,j,n), \forall i \in I, j \in J_i, n \in N}$$

where V_{ij}^{min} and V_{ij}^{max} denotes the minimal and maximal capacity allowed of the specific unit (*j*) when performing task (*i*), respectively.

$$\boxed{B(i,j,n) = V_{ij}^{\max} \cdot wv(i,j,n), i \in I_r, j \in J_i, n \in N}$$

where I_r is the set of reaction tasks.

Duration Constraints

$$\boxed{T^f(i,j,n) = T^s(i,j,n) + \alpha_{ij} wv(i,j,n) + \beta_{ij} B(i,j,n), \forall i \in I, j \in J_i, n \in N}$$

α_{ij}: fixed processing times for reaction and drying tasks, zero for extrusion tasks;

β_{ij}: inverse of processing rates for extrusion tasks, zero for reaction and drying tasks.

Short-Term Scheduling Model

Sequence Constraints

- Same task in the same unit

$$T^s(i,j,n+1) \geq T^f(i,j,n), \forall i \in I, j \in J_i, n \in N, n \neq n_{last}$$

- Different tasks in the same unit

$$T^s(i,j,n+1) \geq T^f(i',j,n) + tcl_{ii'} \cdot wv(i',j,n) - H(1 - wv(i',j,n)),$$
$$\forall j \in J, i \in I_j, i' \in I_j, i \neq i', n \in N, n \neq n_{last}$$

where $tcl_{ii'}$ is the clean-up time for units when switched from task (i) to task (i').

- Different tasks in different units

$$T^s(i,j,n+1) \geq T^f(i',j',n) - H(1 - wv(i',j',n)),$$
$$\forall j, j' \in J, i \in I_j, i' \in I_{j'}, i \neq i', n \in N, n \neq n_{last}$$

Short-Term Scheduling Model

Constraints for Demands with Intermediate Due Dates

$$\boxed{D(s,n) + SL(s,n) = dint_{sn}, \ \forall s \in S, n \in N}$$

where $dint_{sn}$ denotes the demand for state (s) at event point (n).

Due Date Constraints

$$\boxed{T^{s}(i,j,n) \le due_{sn}, \ s \in S, i \in I_s, j \in J_i, n \in N}$$

where due_{sn} is the due time for the demand of state (s) at event point (n).

Constraints for Demands at the End of the Time Horizon

$$\boxed{STF(s) \ge dend_s, \ \forall s \in S}$$

where $dend_s$ is the demand for state (s) at the end of the time horizon.

Short-Term Scheduling Model

Unit Available Time Constraints

$$\boxed{T^s(i,j,n) \geq tav_j - H(1 - wv(i,j,n)), \forall i \in I, j \in J_i, n \in N}$$

where tav_j is the time when unit (j) starts to become available.

Time Horizon Constraints

$$T^f(i,j,n) \leq H, \forall i \in I, j \in J_i, n \in N$$

$$T^s(i,j,n) \leq H, \forall i \in I, j \in J_i, n \in N$$

Objective: Maximization of Production

$$\boxed{-\sum_s \sum_n p_{sn} \cdot SL(s,n) + \gamma \sum_s vd_s \cdot vp_s \cdot vm_s \cdot STF(s)}$$

p_{sn}: priority of the demand for state (s) at event point (n),
vm_s: relative value of state (s) in the material sequence,
vp_s: relative value indicating priority of the corresponding product,
vd_s: relative value indicating importance to fulfill future demands for the corresponding product,
γ: constant coefficient to balance meeting demands with intermediate due dates and overall production.

Robust Optimization: General MILPs

- Lin, X, S.L. Janak and C.A. Floudas, 2004, A New Robust Optimization Approach for Scheduling under Uncertainty: I. Bounded Uncertainty, *Comp. Chem. Engng.* 28, 1069.
- Janak, S.L., X. Lin and C.A. Floudas, 2005, A New Robust Optimization Approach for Scheduling under Uncertainty: II. Uncertainty with Known Distribution, *Comp. Chem. Engng.*, submitted for publication.

Robust Optimization Methodology

- Consider the general class of MILP:

$$
\begin{aligned}
\underset{x,y}{Min / Max} \quad & c^T x + d^T y \\
s.t. \quad & Ex + Fy = e \\
& Ax + By \leq p \\
& x^L \leq x \leq x^U \\
& y_k = \{0,1\}, \ \forall k
\end{aligned}
$$

Assume that both the coefficients and the right-hand-side parameters of the inequality constraint are uncertain, (*i.e.* a_{ij}, b_{ik}, and p_i).

- We are concerned about feasibility of the following inequality:

$$\sum_j a_{ij} x_j + \sum_k b_{ik} y_k \leq p_i$$

Robust Optimization Methodology

- We are concerned about feasibility of the following inequality:

$$\boxed{\sum_j a_{ij} x_j + \sum_k b_{ik} y_k \le p_i}$$

- By introducing a small number of auxiliary variables and applying developments in probability theory, the stochastic problem is converted to its deterministic robust counterpart which gives "reliable" solutions for a given level of uncertainty, infeasibility tolerance, and reliability level when a probabilistic measure is applied.

Bounded Uncertainty

$$|\tilde{a}_{ij} - a_{ij}| \leq \varepsilon |a_{ij}|, |\tilde{b}_{ik} - b_{ik}| \leq \varepsilon |b_{ik}|, |\tilde{p}_i - p_i| \leq \varepsilon |p_i|$$

$\tilde{a}_{ij}, \tilde{b}_{ik}, \tilde{p}_i$: true values; a_{ij}, b_{ik}, p_i : nominal values; ε : uncertainty level

Property 1: Interval Robust Counterpart [ε,δ] (IRC[ε,δ]) – MILP:

$$\begin{aligned}
\underset{x,y,u}{Min/Max} \quad & c^T x + d^T y \\
s.t. \quad & Ex + Fy = e \\
& Ax + By \leq p \\
& \sum_j a_{ij} x_j + \varepsilon \sum_{j \in J_i} |a_{ij}| u_j + \sum_k b_{ik} y_k + \varepsilon \sum_{k \in K_i} |b_{ik}| y_k \\
& \leq p_i - \varepsilon |p_i| + \delta \max[1, |p_i|], \forall i \\
& -u_j \leq x_j \leq u_j, \quad \forall j \\
& x^L \leq x \leq x^U \\
& y_k = \{0, 1\}, \quad \forall k
\end{aligned}$$

δ : infeasibility tolerance level

Bounded and Symmetric Uncertainty

$$\tilde{a}_{ij} = (1+\varepsilon\xi_{ij})\,a_{ij},\ \ \tilde{b}_{ik} = (1+\varepsilon\xi_{ik})\,b_{ik},\ \ \tilde{p}_i = (1+\varepsilon\xi_i)\,p_i$$

$\xi_{ij},\ \xi_{ik},\ \xi_i$: random variables distributed symmetrically in [-1,1].

$$P\{\sum_j \tilde{a}_{ij} x_j + \sum_k \tilde{b}_{ik} y_k > \tilde{p}_i + \delta \max[1, |\, p_i \,|]\} \le \kappa$$

κ : reliability level.

Property 2: Robust Counterpart [ε,δ,κ] (RC[ε,δ,κ]) – Convex MINLP:

$$\sum_j a_{ij} x_j + \sum_k b_{ik} y_k + \varepsilon[\sum_{j\in J_i} |\, a_{ij} \,|\, u_{ij} + \Omega \sqrt{\sum_{j\in J_i} a_{ij}^2 z_{ij}^2 + \sum_{k\in K_i} b_{ik}^2 y_k + p_i^2}$$
$$\le p_i + \delta \max[1, |\, p_i \,|], \ \forall i$$
$$-u_{ij} \le x_j - z_{ij} \le u_{ij},\ \ \forall i, j$$

where $\kappa = exp\{-\Omega^2 / 2\}$.

Uncertainty with Known Distribution

$$\tilde{a}_{ij} = (1+\varepsilon\xi_{ij})\,a_{ij},\ \ \tilde{b}_{ik} = (1+\varepsilon\xi_{ik})\,b_{ik},\ \ \tilde{p}_i = (1+\varepsilon\xi_i)\,p_i$$

$\xi_{ij},\ \xi_{ik},\ \xi_i$: random variables of known distribution.

Define:

$$\xi = \sum_{j\in J_i}\xi_{ij}\left|a_{ij}\right|x_j + \sum_{k\in K_i}\xi_{ik}\left|b_{ik}\right|y_k - \xi_i\left|p_i\right|$$

Cumulative distribution function:

$$F_\xi(\lambda) = P\{\xi \le \lambda\} = 1 - P\{\xi > \lambda\} = 1 - \kappa$$

Inverse distribution function (quantile function):

$$F_\xi^{-1}(1-\kappa) = f\left(\lambda, \left|a_{ij}\right|x_j, \left|b_{ik}\right|y_k, \left|p_i\right|\right)$$

Robust Counterpart[ε,δ,κ] (RC[ε,δ,κ]) – MILP or Convex MINLP:

$$\sum_j a_{ij}x_j + \sum_k b_{ik}y_k + \varepsilon\cdot f\left(\lambda, \left|a_{ij}\right|x_j, \left|b_{ik}\right|y_k, \left|p_i\right|\right) \le p_i + \delta\max\left[1, \left|p_i\right|\right],\ \forall i$$

Uncertainty with Uniform Distribution

One uncertain parameter per constraint:

$$\boxed{\tilde{b}_{ik} = (1 + \varepsilon \xi_{ik})\, b_{ik}}$$

ξ_{ik} : random variable of uniform distribution in [-1,1].

Property 3: Robust Counterpart [ε,δ,κ] – MILP:

$$
\begin{aligned}
\min_{x,y} / Max \quad & c^T x + d^T y \\
s.t. \quad & Ex + Fy = e \\
& Ax + By \le p \\
& \sum_j a_{ij} x_j + \sum_k b_{ik} y_k + \varepsilon (1 - 2\kappa) |b_{ik}| y_k \\
& \le p_i + \delta \max[1, |p_i|], \ \forall i \\
& x^L \le x \le x^U \\
& y_k = \{0, 1\}, \ \forall k
\end{aligned}
$$

Uncertainty with Normal Distribution

Property 4: Robust Counterpart [ε,δ,κ] – Convex MINLP:

$$
\begin{aligned}
\underset{x,y}{Min/Max} \quad & c^T x + d^T y \\
s.t. \quad & Ex + Fy = e \\
& Ax + By \le p \\
& \sum_j a_{ij} x_j + \sum_k b_{ik} y_k + \varepsilon\lambda \sqrt{\sum_{j\in J_i} a_{ij}^2 x_j^2 + \sum_{k\in K_i} b_{ik}^2 y_k + p_i^2} \\
& \le p_i + \delta \max[1, |p_i|], \forall i \\
& x^L \le x \le x^U \\
& y_k = \{0, 1\}, \ \forall k
\end{aligned}
$$

where $\lambda = F^{-1}_n(1-\kappa)$ and F^{-1}_n is the inverse distribution function of a standardized normal distribution.

Uncertainty with a Difference of Normal Distributions

Property 5: Robust Counterpart [ε,δ,κ] – Convex MINLP:

$$
\begin{aligned}
\min_{x,y} / \max_{x,y} \quad & c^T x + d^T y \\
\text{s.t.} \quad & Ex + Fy = e \\
& Ax + By \le p \\
& \sum_j a_{ij} x_j + \sum_k b_{ik} y_k + \varepsilon \lambda \sqrt{\sum_{j \in J_i} a_{ij}^2 x_j^2 + \sum_{k \in K_i} b_{ik}^2 y_k + p_i^2} \\
& \le p_i + \delta \max[1, |p_i|], \ \forall i \\
& x^L \le x \le x^U \\
& y_k = \{0, 1\}, \ \forall k
\end{aligned}
$$

where $\lambda = F_n^{-1}(1-\kappa)$ and F_n^{-1} is the inverse distribution function of a standardized normal distribution, and μ and σ are the mean and standard deviation, respectively, of the difference of two normal random variables.

Uncertainty with Discrete Distribution

Property 6: Robust Counterpart [ε,δ,κ] – MILP:

$$
\begin{aligned}
\underset{x,y}{Min/Max} \quad & c^T x + d^T y \\
s.t. \quad & Ex + Fy = e \\
& Ax + By \le p \\
& \sum_j a_{ij} x_j + \sum_k b_{ik} y_k + \varepsilon \lambda \\
& \le p_i + \delta \max[1, |p_i|], \forall i \\
& x^L \le x \le x^U \\
& y_k = \{0, 1\}, \ \forall k
\end{aligned}
$$

where $\lambda = F^{-1}_n(1-\kappa)$ and F^{-1}_n is the inverse distribution function of the overall discrete distribution of the sum of several discrete random variables,

$$\xi = \sum_{j \in J_i} \xi_{ij} |a_{ij}| x_j + \sum_{k \in K_i} \xi_{ik} |b_{ik}| y_k - \xi_i |p_i|.$$

Uncertainty with Binomial Distribution

Property 7: Robust Counterpart [ε,δ,κ] – MILP:

$$
\begin{aligned}
\underset{x,y}{Min/Max} \quad & c^T x + d^T y \\
s.t. \quad & Ex + Fy = e \\
& Ax + By \le p \\
& \sum_j a_{ij} x_j + \sum_k b_{ik} y_k + \varepsilon\lambda |a_{ij'}| x_{j'} \\
& \le p_i + \delta \max[1, |p_i|], \forall i \\
& x^L \le x \le x^U \\
& y_k = \{0, 1\}, \quad \forall k
\end{aligned}
$$

where we consider only a single uncertain parameter per constraint and $\lambda = F^{-1}_b(1-\kappa)$ and F^{-1}_b is the inverse distribution function of a binomial distribution with parameters *n* and *p*.

Uncertainty with Poisson Distribution

Property 8: Robust Counterpart [ε,δ,κ] – MILP:

$$\begin{aligned}
\underset{x,y}{Min/Max} \quad & c^T x + d^T y \\
s.t. \quad & Ex + Fy = e \\
& Ax + By \le p \\
& \sum_j a_{ij} x_j + \sum_k b_{ik} y_k + \varepsilon\lambda |a_{ij'}| x_{j'} \\
& \le p_i + \delta \max[1, |p_i|], \forall i \\
& x^L \le x \le x^U \\
& y_k = \{0, 1\}, \ \forall k
\end{aligned}$$

where we consider only a single uncertain parameter per constraint and $\lambda = F^{-1}_p(1-\kappa)$ and F^{-1}_p is the inverse distribution function of a binomial distribution with parameter γ.

Continuous-Time Formulation for Short-Term Scheduling

Ierapetritou and Floudas (1998a,b); Janak and Floudas (2004)
Floudas and Lin (2004), (2005); Lin and Floudas (2001)

- Unit-specific continuous-time representation
- Mixed Integer Linear Programming (MILP) Problem

Max/Min	Profit/Make-span
s.t.	Allocation constraints
	Capacity constraints
	Material balance constraints
	Storage constraints
	Demand constraints
	Duration constraints
	Sequence constraints
	Time horizon constraints

Uncertainty in Processing Times

$$\boxed{\begin{aligned} T^{f}_{i,j,n} - T^{s}_{i,j,n} &= \tilde{\alpha} \cdot wv_{i,n} + \tilde{\beta} \cdot b_{i,j,n} \\ T^{f}_{i,j,n} - T^{s}_{i,j,n} &\geq \alpha \cdot wv_{i,n} + \beta \cdot b_{i,j,n} \end{aligned}}$$

- Bounded Uncertainty: $\boxed{\alpha^{L} \leq \tilde{\alpha} \leq \alpha^{U}, \quad \beta^{L} \leq \tilde{\beta} \leq \beta^{U}}$

IRC[ε,δ]: (MILP) - $\boxed{T^{f}_{i,j,n} - T^{s}_{i,j,n} \geq \alpha^{U} \cdot wv_{i,n} + \beta^{U} \cdot b_{i,j,n} - \delta}$

- Uncertainty with Known Distribution (β = 0): $\boxed{\tilde{\alpha} = (1 + \varepsilon \xi_{\alpha})\alpha}$
  - Uniform Distribution in [-1,1]:

RC[ε,δ,κ]: (MILP) – $\boxed{T^{s}_{i,j,n} - T^{f}_{i,j,n} + (1 + \varepsilon(1 - 2\kappa))\alpha \cdot wv_{i,n} \leq \delta}$

  - Normal Distribution:

RC[ε,δ,κ]: (MILP) – $\boxed{T^{s}_{i,j,n} - T^{f}_{i,j,n} + (1 + \varepsilon\lambda)\alpha \cdot wv_{i,n} \leq \delta}$

where $\lambda = F^{-1}_{n}(1-\kappa)$ and F^{-1}_{n} is the inverse distribution function of a standardized normal distribution.

Uncertainty in Processing Times

- Difference of Normal Distributions:

RC[ε,δ]: (MILP) – $\boxed{T^{s}_{i,j,n} - T^{f}_{i,j,n} + (1 + \varepsilon[\lambda\sqrt{\sigma} + \mu])\alpha \cdot wv_{i,n} \leq \delta}$

where $\lambda = F^{-1}_{n}(1-\kappa)$ and F^{-1}_{n} is the inverse distribution function of a standardized normal distribution, and μ and σ are the mean and standard deviation, respectively, of the difference of two normal random variables.

- Discrete Distribution (General, Binomial, or Poisson):

RC[ε,δ,κ]: (MILP) – $\boxed{T^{s}_{i,j,n} - T^{f}_{i,j,n} + (1 + \varepsilon\lambda)\alpha \cdot wv_{i,n} \leq \delta}$

where $\lambda = F^{-1}_{d}(1-\kappa)$ and F^{-1}_{d} is the inverse distribution function of a general discrete distribution or a binomial or poisson distribution.

Uncertainty in Product Demands

$$\boxed{STF(s) \geq \widetilde{dem}_s - \delta}$$

- Bounded Uncertainty: $\boxed{dem_s^L \leq \widetilde{dem}_s \leq dem_s^U}$

IRC: (MILP) – $\boxed{STF(s) \geq dem_s^U - \delta \cdot dem_s}$

- Uncertainty with Known Distribution: $\boxed{\widetilde{dem}_s = (1 + \varepsilon \xi_s) dem_s}$
  - Uniform Distribution in [-1,1]:

RC[$\varepsilon,\delta,\kappa$]: (MILP) – $\boxed{STF(s) \geq (1 + \varepsilon(1 - 2\kappa) - \delta) \cdot dem_s}$

  - Normal Distribution:

RC[$\varepsilon,\delta,\kappa$]: (MILP) – $\boxed{STF(s) \geq (1 + \varepsilon\lambda - \delta) \cdot dem_s}$

where $\lambda = F^{-1}_n(1-\kappa)$ and F^{-1}_n is the inverse distribution function of a standardized normal distribution.

Uncertainty in Product Demands

- Difference of Normal Distributions:

  RC[ε,δ,κ]: (MILP) – $\boxed{STF(s) \geq (1 + \varepsilon[\lambda\sqrt{\sigma} + \mu] - \delta) \cdot dem_s}$

  where $\lambda = F^{-1}_n(1-\kappa)$ and F^{-1}_n is the inverse distribution function of a standardized normal distribution, and μ and σ are the mean and standard deviation, respectively, of the difference of two normal random variables.

- Discrete Distribution (General, Binomial, or Poisson):

  RC[ε,δ,κ]: (MILP) – $\boxed{STF(s) \geq (1 + \varepsilon\lambda - \delta) \cdot dem_s}$

  where $\lambda = F^{-1}_d(1-\kappa)$ and F^{-1}_d is the inverse distribution function of a general discrete distribution or a binomial or poisson distribution.

Uncertainty in Market Prices

$$\begin{aligned} &\textit{Maximize} \quad \text{Profit} \\ &s.t. \quad \text{Profit} \le \sum_{s\in S_p} \tilde{p}_s \cdot STF(s) - \sum_{s\in S_r} \tilde{p}_s \cdot STI(s) \end{aligned}$$

- Bounded Uncertainty: $p_s^L \le \tilde{p}_s \le p_s^U$

IRC: (MILP) – $\text{Profit}(1-\delta) \le \sum_{s\in S_p} p_s^U \cdot STF(s) - \sum_{s\in S_r} p_s^U \cdot STI(s)$

- Bounded and Symmetric Uncertainty in [-1,1]:

RC[ε,δ,κ]: (Convex MINLP) –

$$\sum_{s\in S_r} p_s \cdot STI(s) - \sum_{s\in S_p} p_s \cdot STF(s) + \text{Profit}(1-\delta)$$
$$\varepsilon\left[\sum_{s\in S_r} p_s y(s) + \sum_{s\in S_p} p_s y(s) + \Omega\sqrt{\sum_{s\in S_r} p_s^2 z(s)^2 + \sum_{s\in S_p} p_s^2 z(s)^2}^{\,2}\right] \le 0$$
$$-y(s) \le STI(s) - z(s) \le y(s), \ \ \forall s \in S_r$$
$$-y(s) \le STF(s) - z(s) \le y(s), \ \ \forall s \in S_p$$

where $\kappa = \exp\{-\Omega^2/2\}$.

Uncertainty in Market Prices

- Normal Distribution:

  RC[ε,δ,κ]: (Convex MINLP) –

$$\boxed{\begin{aligned}&\sum_{s\in S_r} p_s STI(s) - \sum_{s\in S_p} p_s STF(s) + \text{Profit}(1-\delta)\\&+\varepsilon\lambda\sqrt{\sum_{s\in S_p} p_s^2 STF(s)^2 + \sum_{s\in S_r} p_s^2 STI(s)^2} \le 0\end{aligned}}$$

  where $\lambda = F^{-1}_n(1-\kappa)$ and F^{-1}_n is the inverse distribution function of a standardized normal distribution.

- Difference of Normal Distributions:

  RC[ε,δ,κ]: (MILP) –

$$\boxed{\sum_{s\in S_r} p_s STI(s) - \sum_{s\in S_p} p_s STF(s) + \text{Profit}(1-\delta) + \varepsilon[\lambda\sqrt{\sigma} + \mu] \le 0}$$

  where $\lambda = F^{-1}_n(1-\kappa)$ and F^{-1}_n is the inverse distribution function of a standardized normal distribution, and μ and σ are the mean and standard deviation, respectively, of the difference of two normal random variables.

Uncertainty in Market Prices

- Discrete Distribution (General, Binomial, or Poisson):

  RC[ε,δ,κ]: (MILP) –

$$\boxed{\sum_{s\in S_r} p_s STI(s) - \sum_{s\in S_p} p_s STF(s) + \text{Profit}(1-\delta) + \varepsilon\lambda \le 0}$$

  where $\lambda = F^{-1}_d(1-\kappa)$ and F^{-1}_d is the inverse distribution function of the overall discrete distribution of the sum of several discrete random variables,

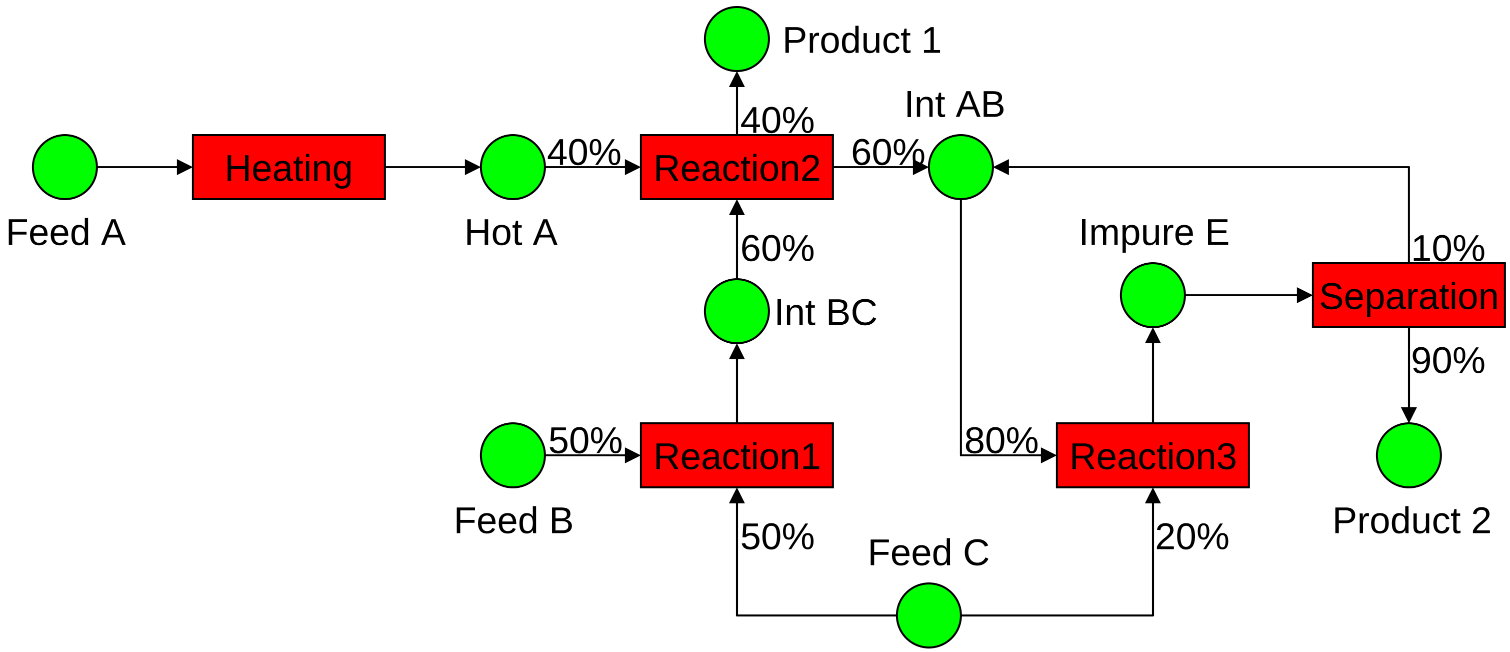
$$\sum_{S_p} \xi_s |p_s| STF(s) + \sum_{S_r} \xi_s |p_s| STI(s)$$

  or a binomial or a poisson distribution of a single random variable.

Robust Optimization for Scheduling under Uncertainty

	Processing Time	Demand	Price
Constraint	$T^f - T^s \geq \alpha \cdot wv + \beta \cdot b$	$STF(s) \geq dem_s$	$P \leq \sum_s p_s ST(s)$
Bounded	MILP	MILP	MILP
Bounded and Symmetric	------	------	convex MINLP
Uniform Distribution	MILP	MILP	------
Normal Distribution	convex MINLP or MILP	MILP	convex MINLP
Difference of Normal Distributions	convex MINLP or MILP	MILP	convex MINLP
Discrete Distribution	MILP	MILP	MILP
Binomial Distribution	MILP	MILP	------
Poisson Distribution	MILP	MILP	------

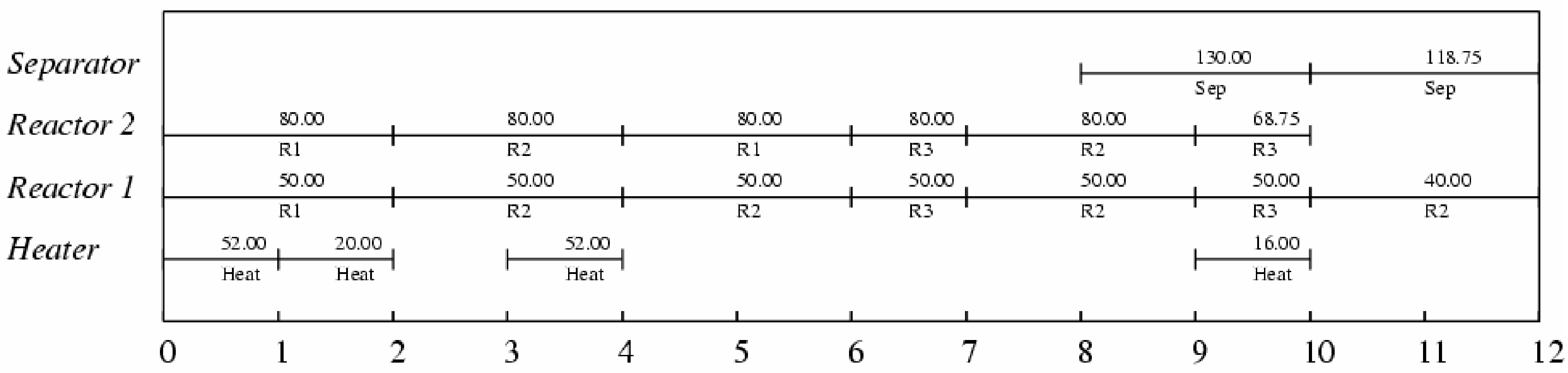
Motivating Example



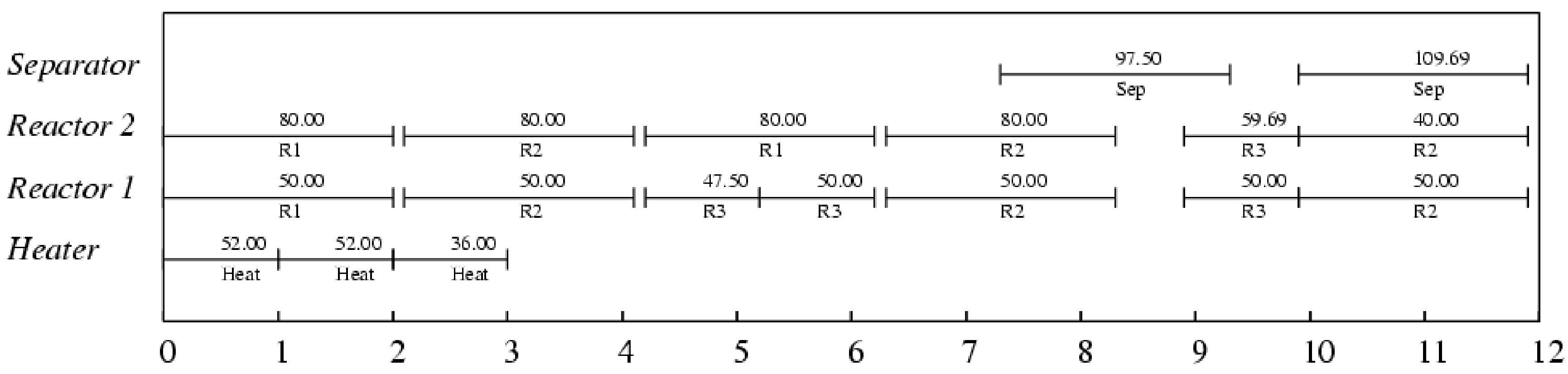
State-Task Network Representation

Units	Capacity	Suitability	Processing Time
Heater	100	Heating	1.0
Reactor 1	50	Reaction1, 2, 3	2.0, 2.0, 1.0
Reactor 2	80	Reaction1, 2, 3	2.0, 2.0, 1.0
Separator	200	Separation	2.0

Motivating Example: Bounded Uncertainty in Processing Time



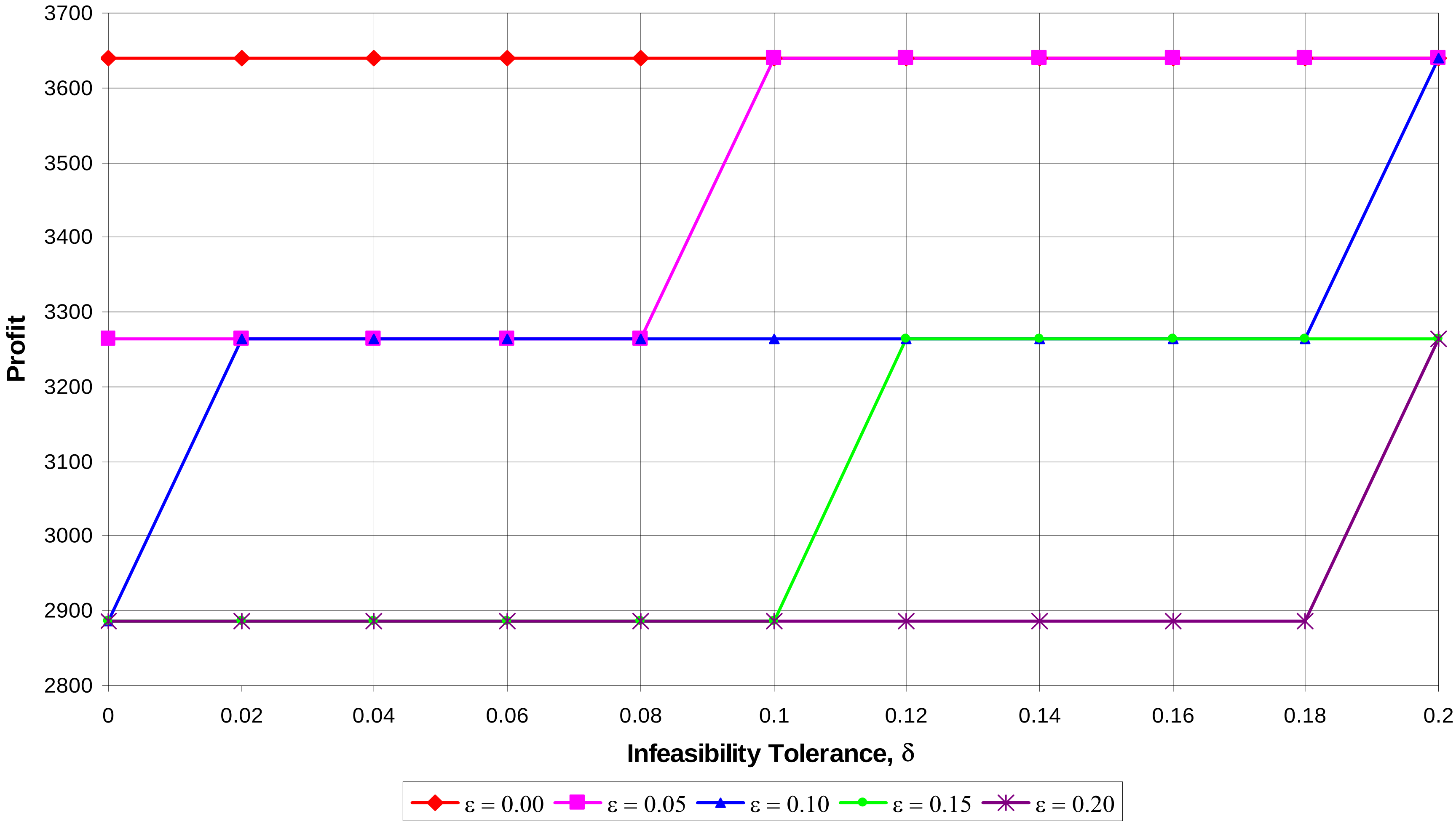
Nominal solution of the motivating example (profit = 3638.75)



Robust solution of the motivating example (profit = 3264.69)

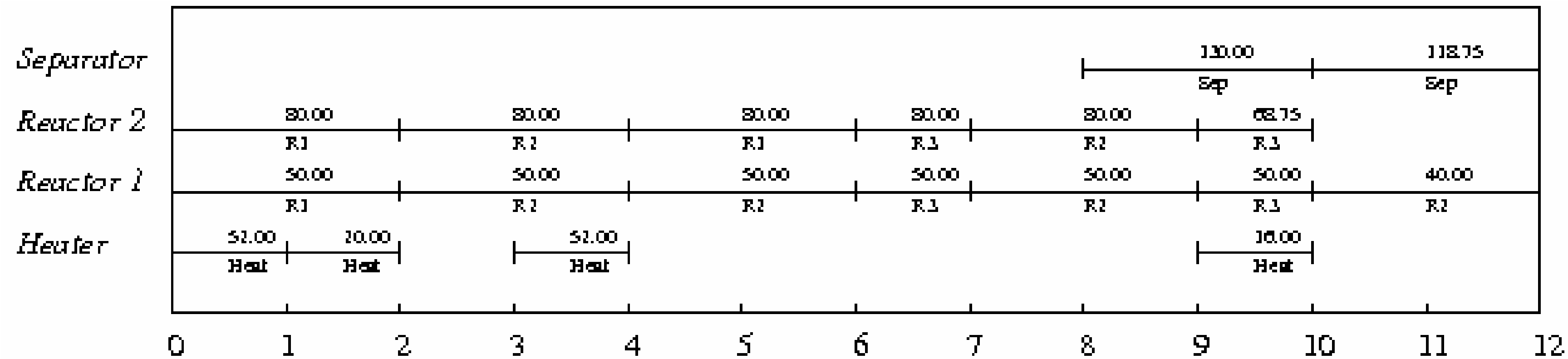
- uncertainty level, $\varepsilon = 0.10$
- infeasibility tolerance level, $\delta = 0.15$

Motivating Example: Bounded Uncertainty in Processing Time

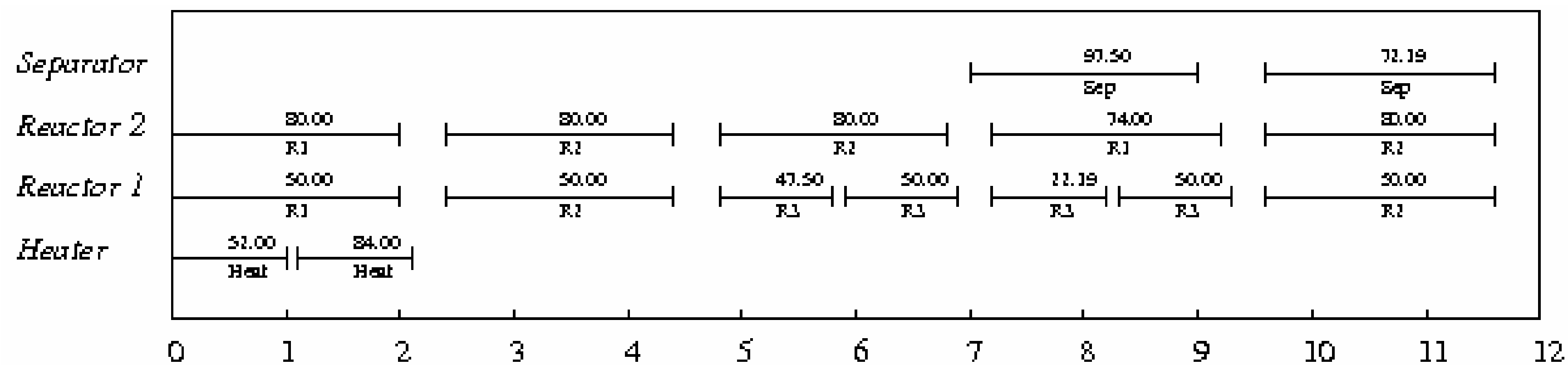


Profit vs. infeasibility tolerance (δ) at different uncertainty levels (ε)

Motivating Example: Poisson Uncertainty in Processing Time



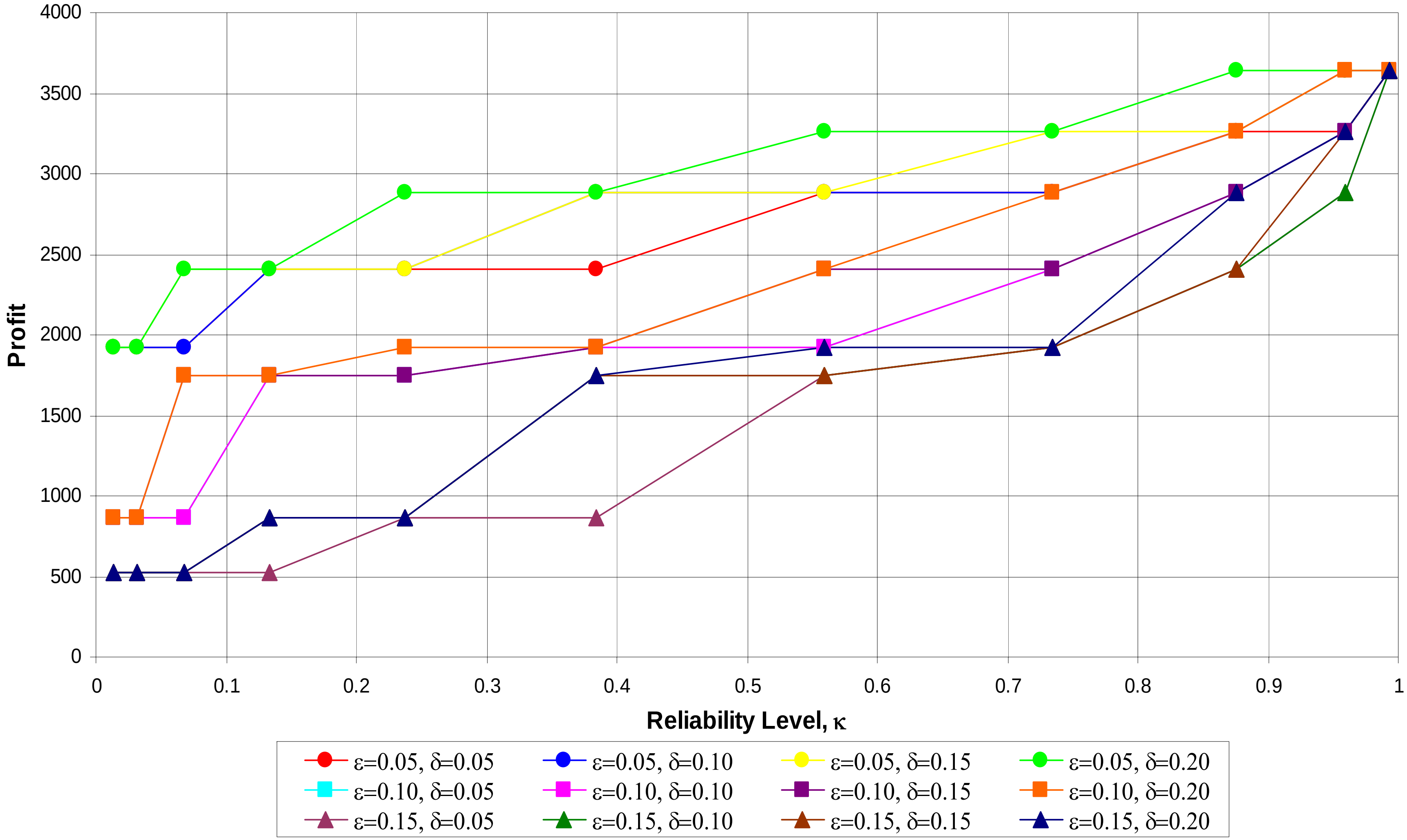
Nominal solution of the motivating example (profit = 3638.75)



Robust solution of the motivating example (profit = 2887.19)

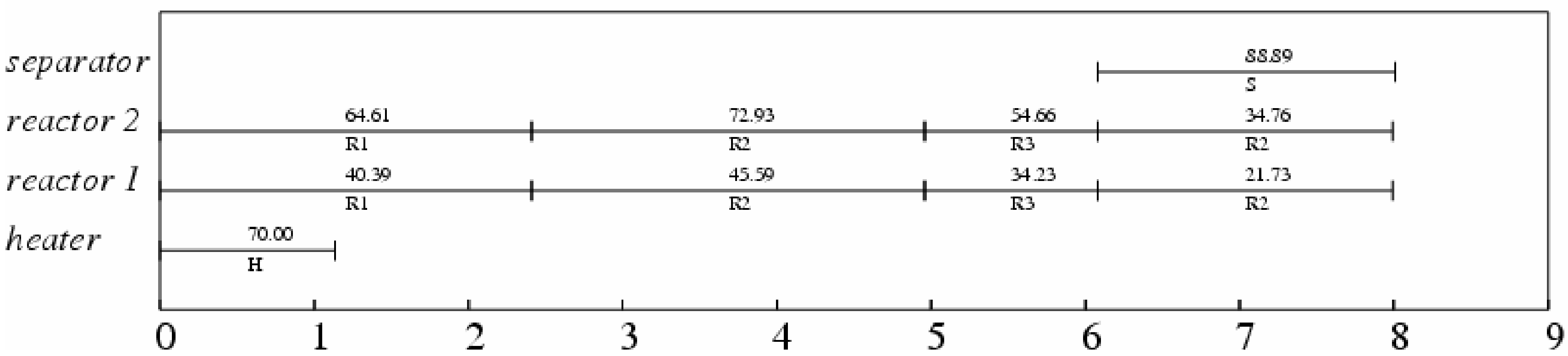
- uncertainty level, $\varepsilon = 0.05$
- infeasibility tolerance level, $\delta = 0.20$
- reliability level, $\kappa = 0.24$

Motivating Example: Poisson Uncertainty in Processing Time

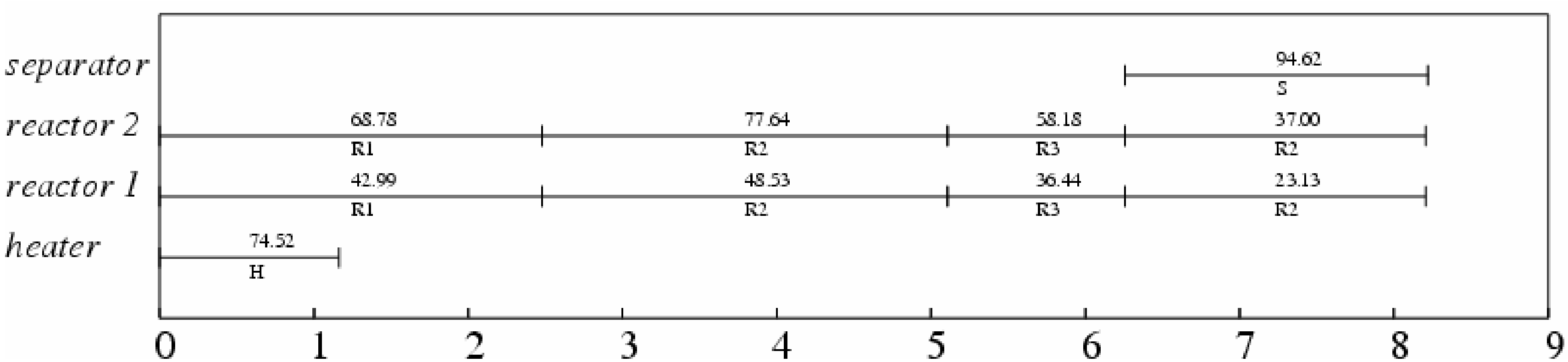


Profit vs. reliability level (κ) at different uncertainty and infeasibility levels.

Motivating Example: Normal Uncertainty in Product Demand



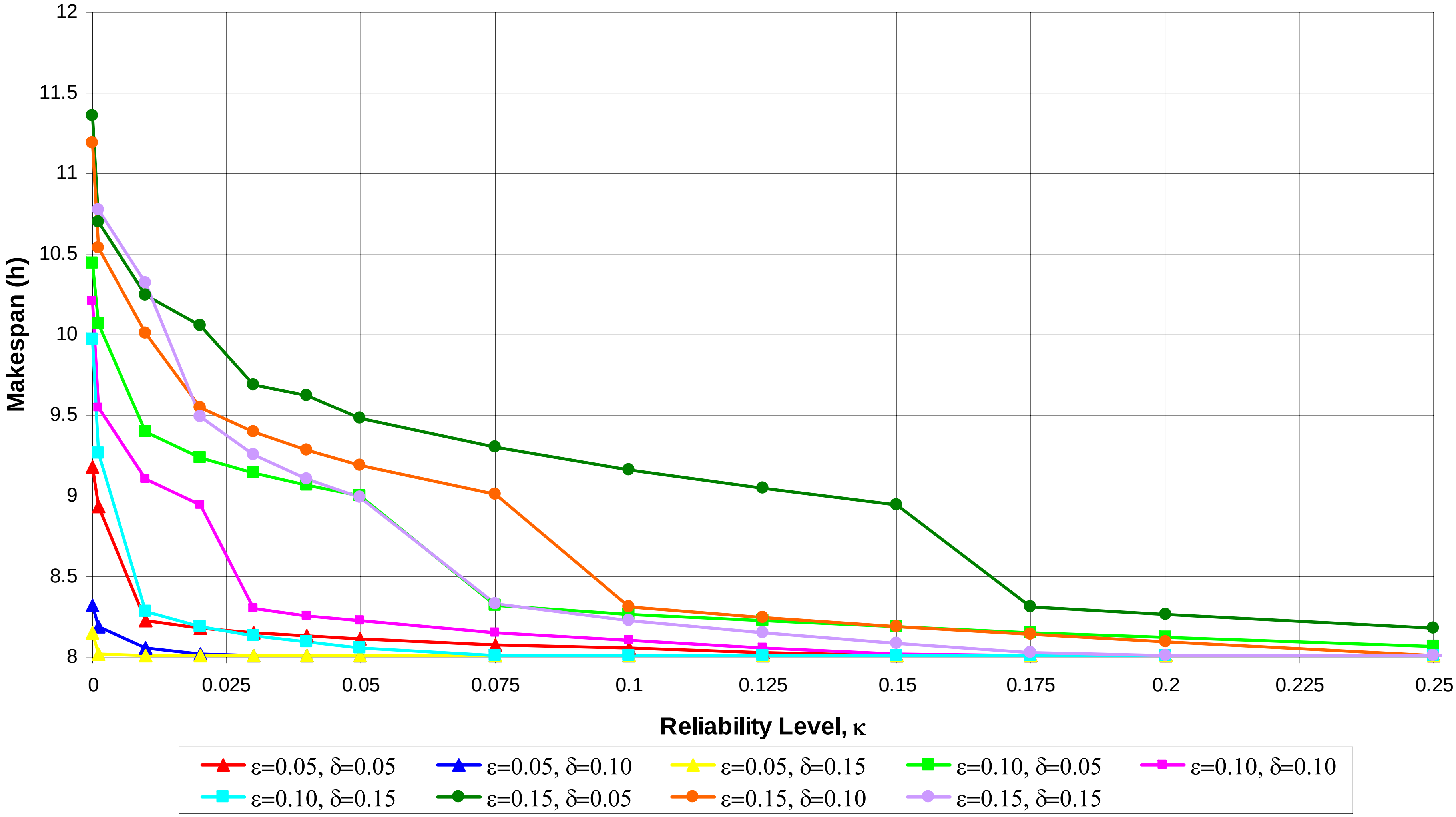
Nominal solution of the motivating example (makespan = 8.007)



Robust solution of the motivating example (makespan = 8.222)

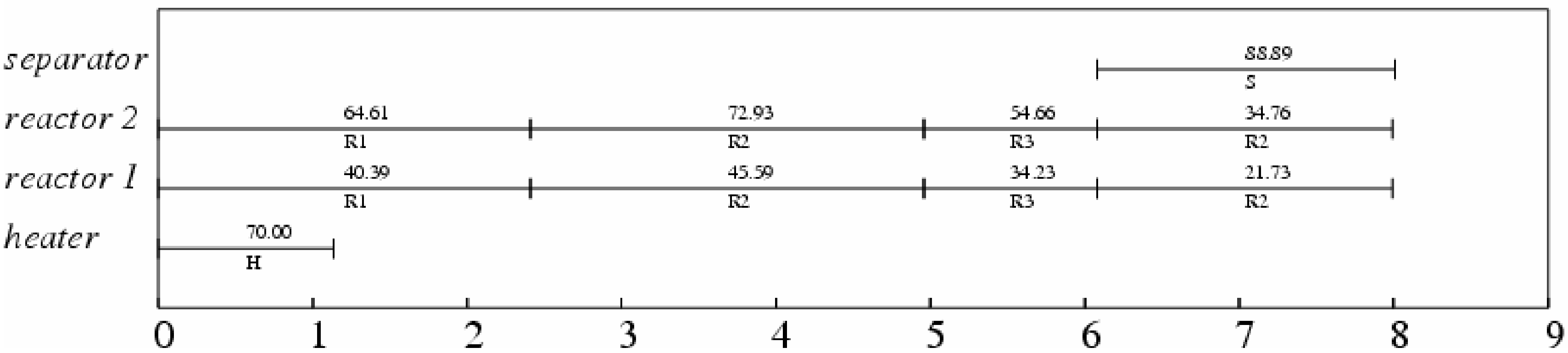
- uncertainty level, $\varepsilon = 0.10$
- infeasibility tolerance level, $\delta = 0.10$
- reliability level, $\kappa = 0.05$

Motivating Example: Normal Uncertainty in Product Demand

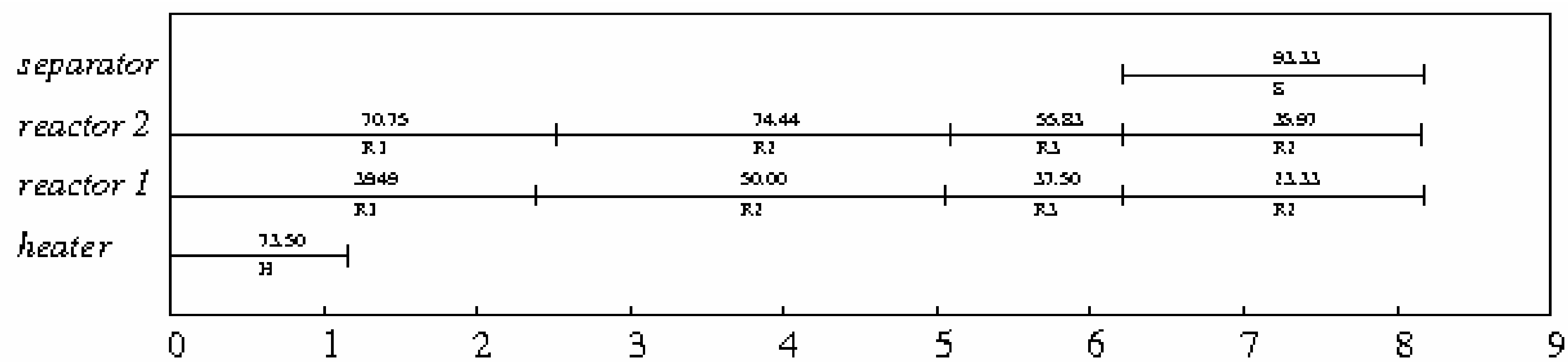


Profit vs. reliability level (κ) at different uncertainty and infeasibility levels.

Motivating Example: Uniform Uncertainty in Product Demand



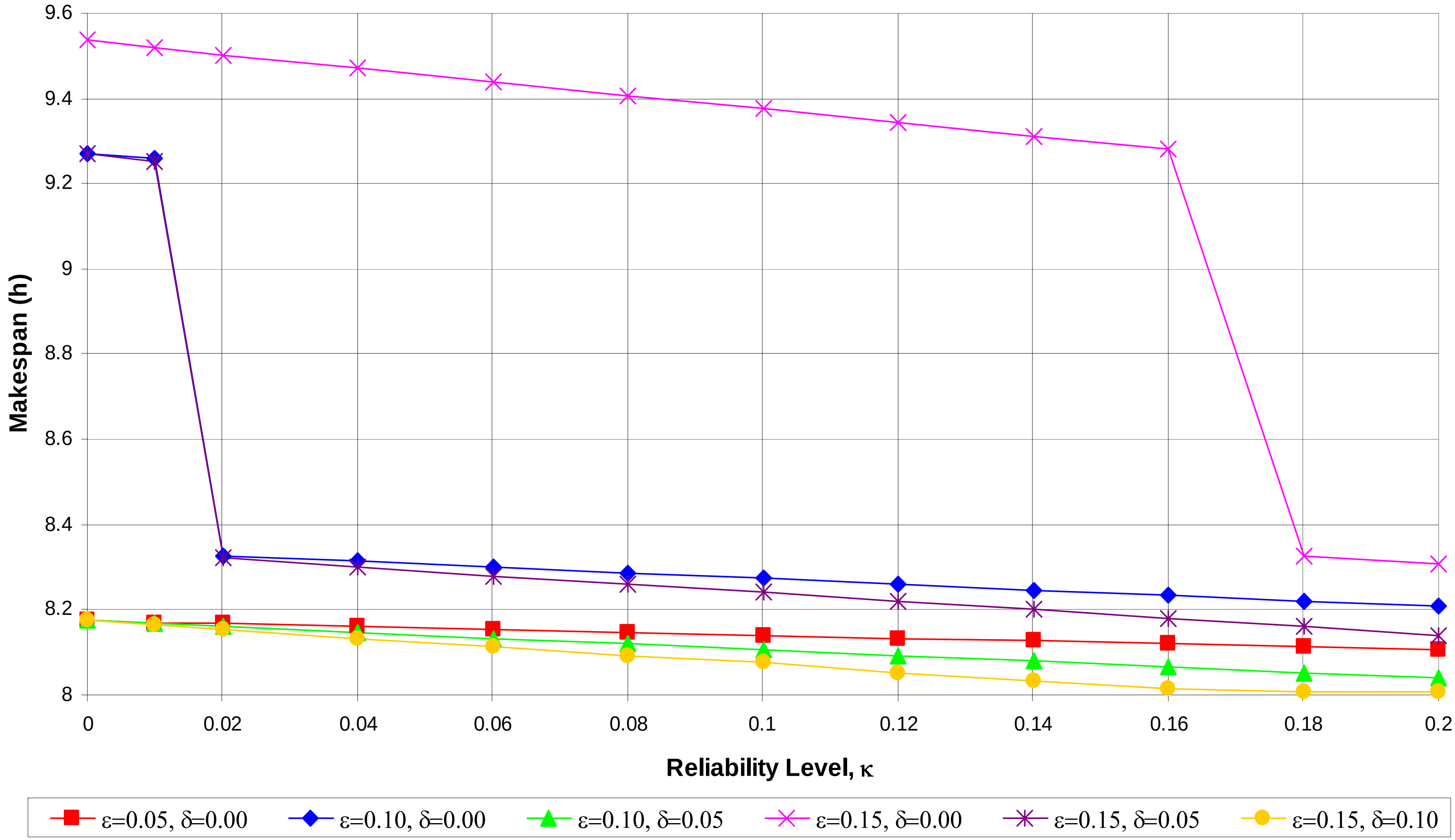
Nominal solution of the motivating example (makespan = 8.007)



Robust solution of the motivating example (makespan = 8.174)

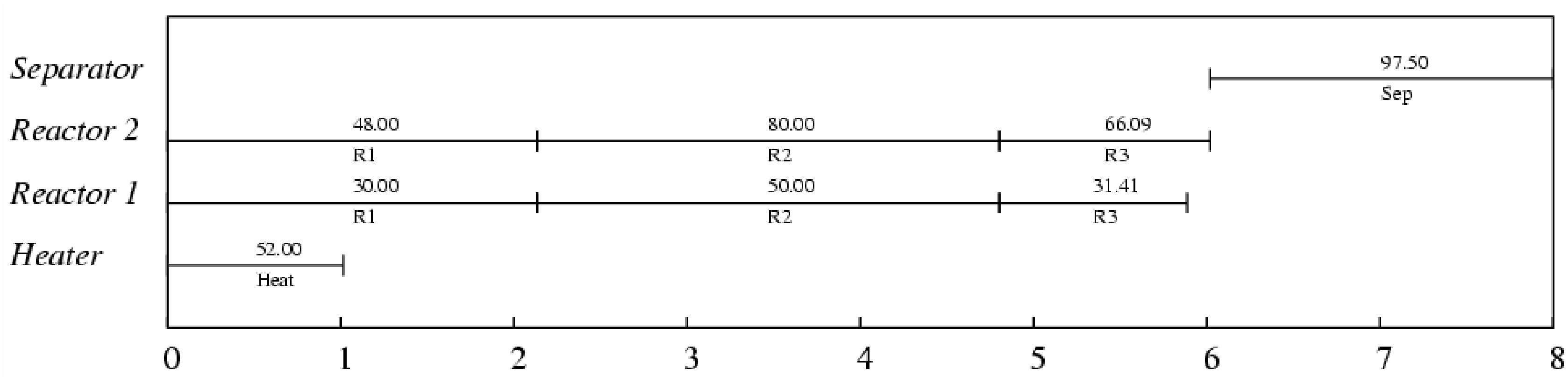
- uncertainty level, ε = 0.10
- infeasibility tolerance level, δ = 0.05
- reliability level, κ = 0.00

Motivating Example: Uniform Uncertainty in Product Demand

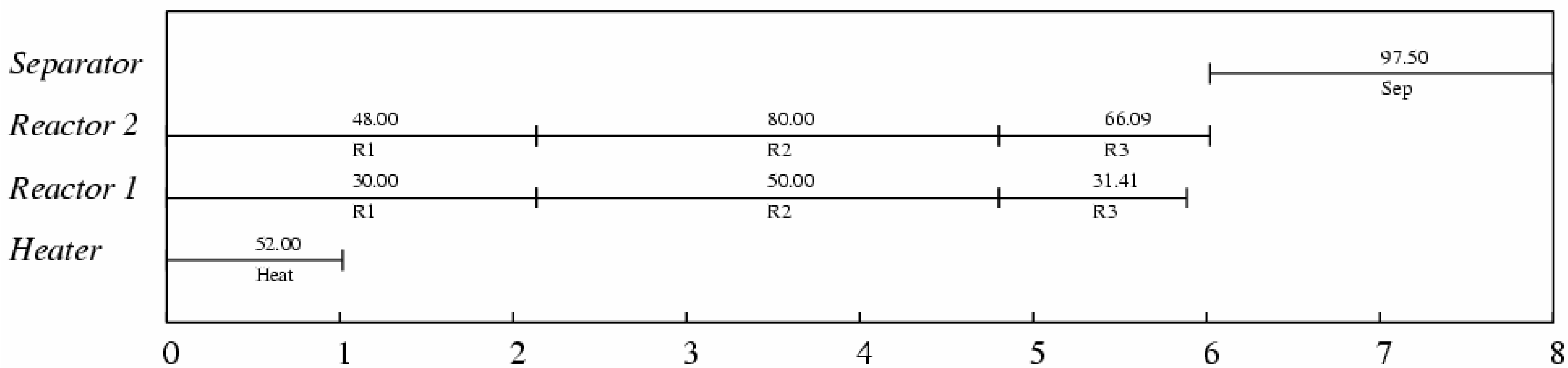


Profit vs. reliability level (κ) at different uncertainty and infeasibility levels.

Motivating Example: Bounded and Symmetric Uncertainty in Market Prices



Nominal solution of the motivating example (profit = 1088.75)



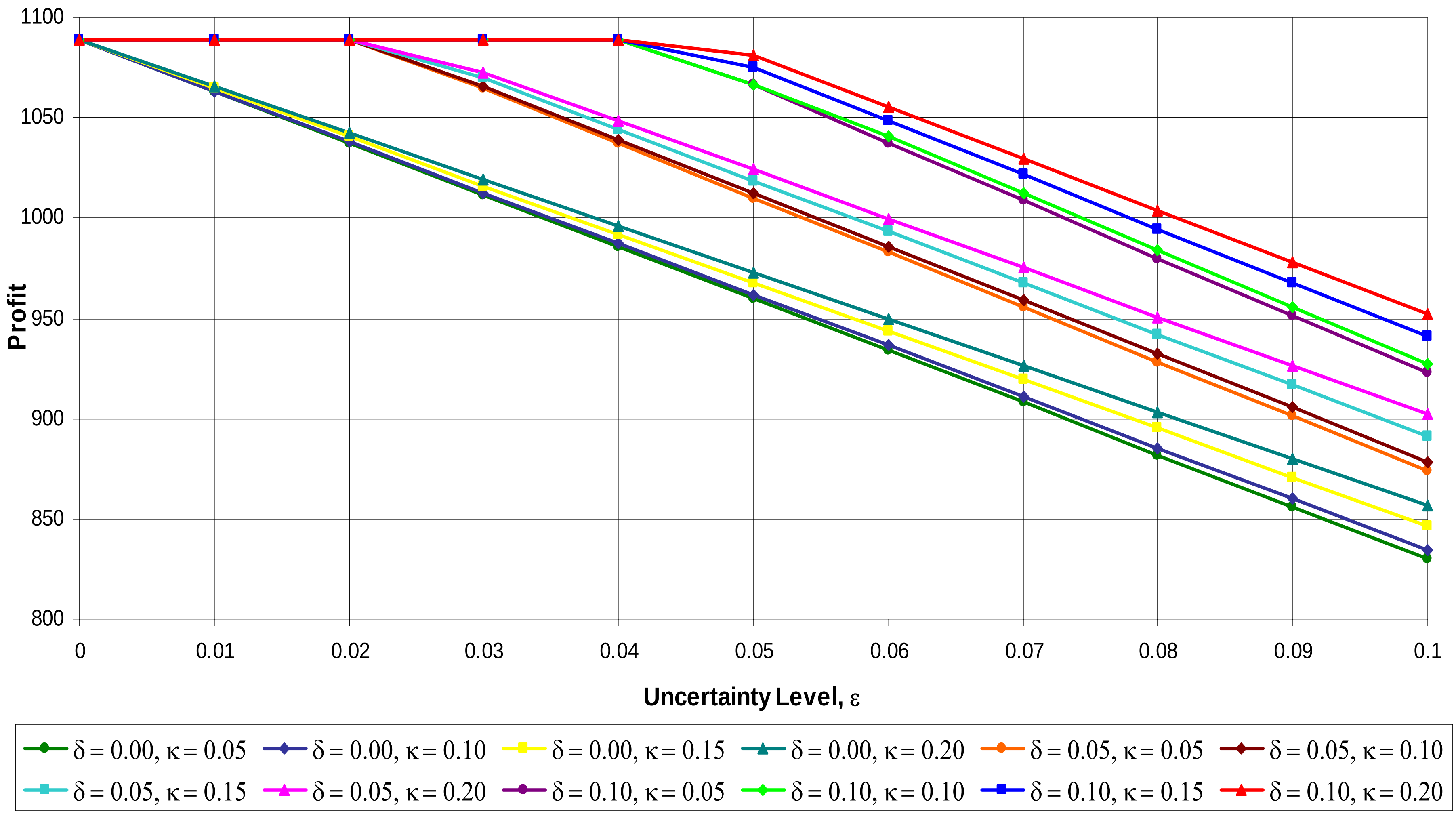
Robust solution of the motivating example (profit = 955.23)

uncertainty level, $\varepsilon = 0.05$

infeasibility tolerance level, $\delta = 0.05$

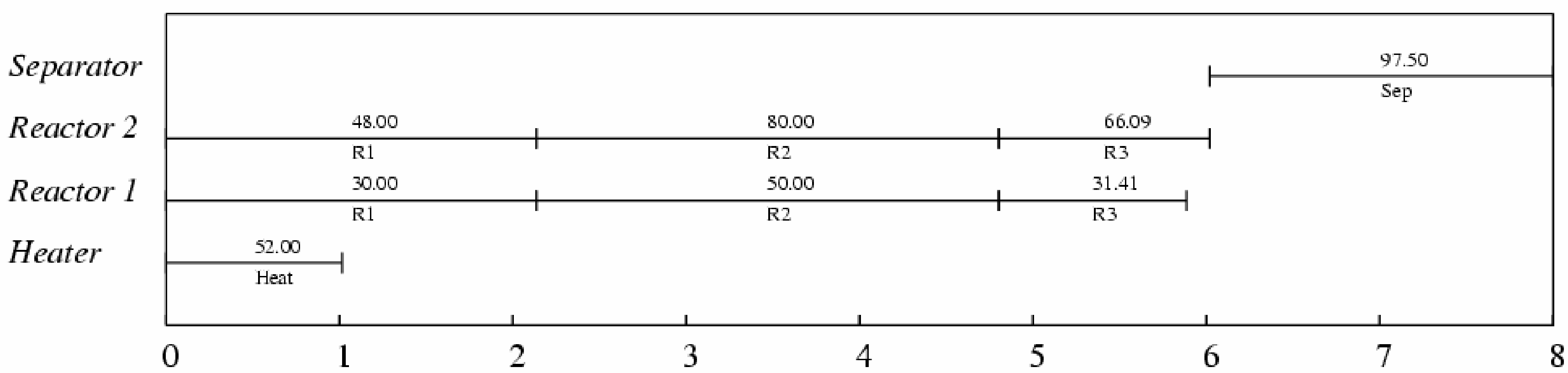
reliability level, $\kappa = 0.05$

Motivating Example: Bounded and Symmetric Uncertainty in Market Prices

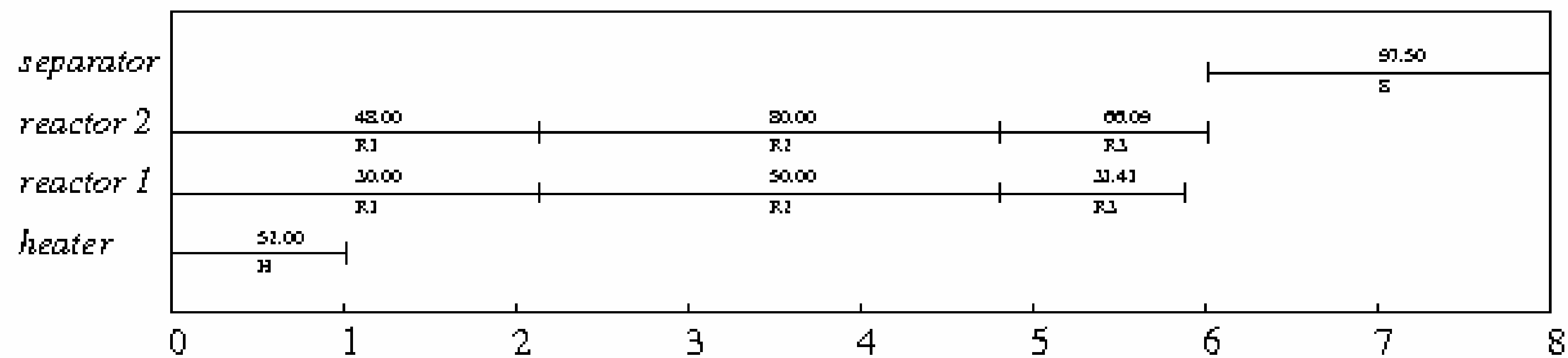


Profit vs. uncertainty level (ε) at different infeasibility and reliability levels.

Motivating Example: Normal Uncertainty in Market Prices



Nominal solution of the motivating example (profit = 1088.75)



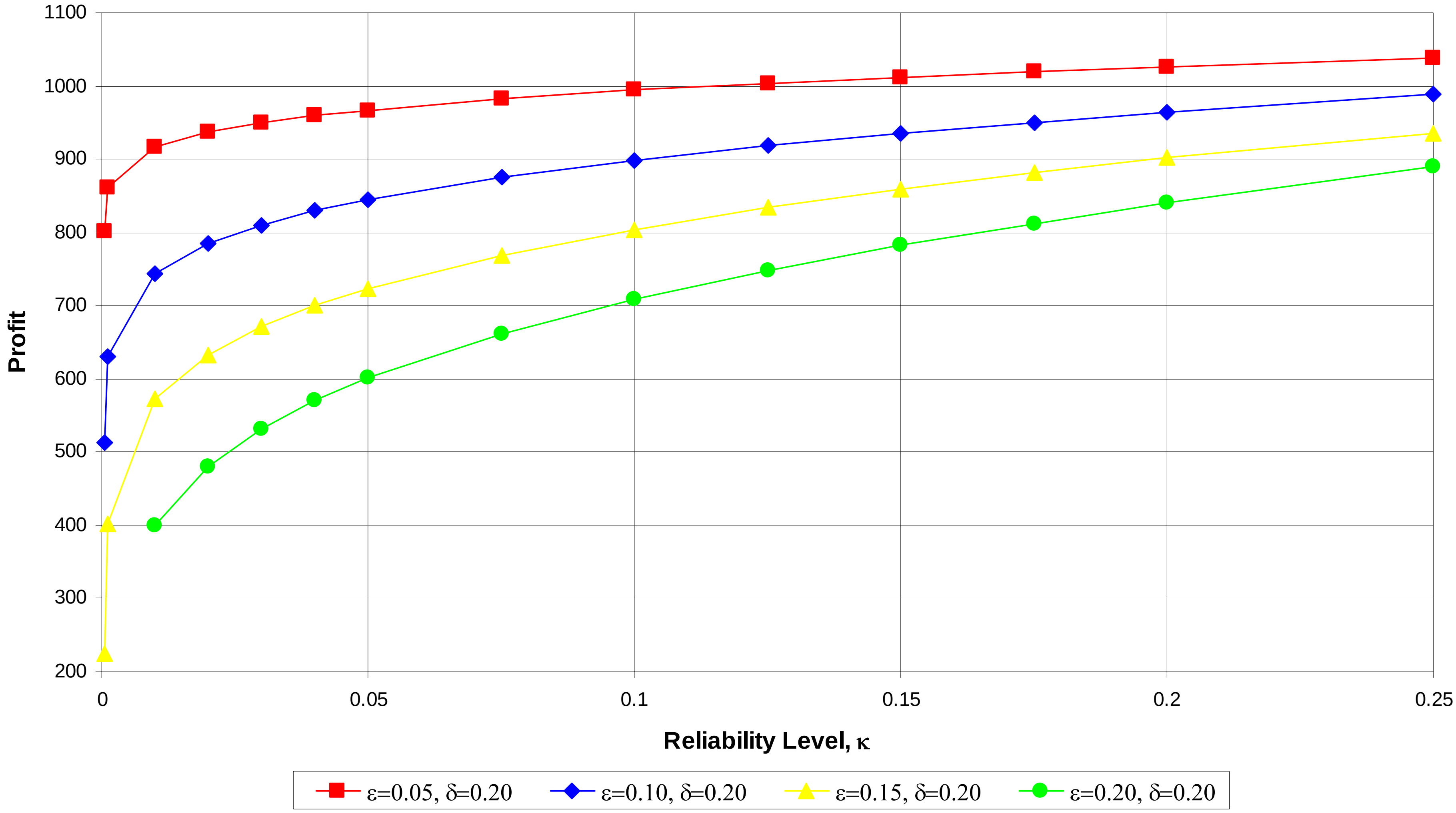
Robust solution of the motivating example (profit = 966.97)

uncertainty level, ε = 0.05

infeasibility tolerance level, δ = 0.05

reliability level, κ = 0.05

Motivating Example: Normal Uncertainty in Market Prices



Profit vs. reliability level (κ) at different uncertainty and infeasibility levels.

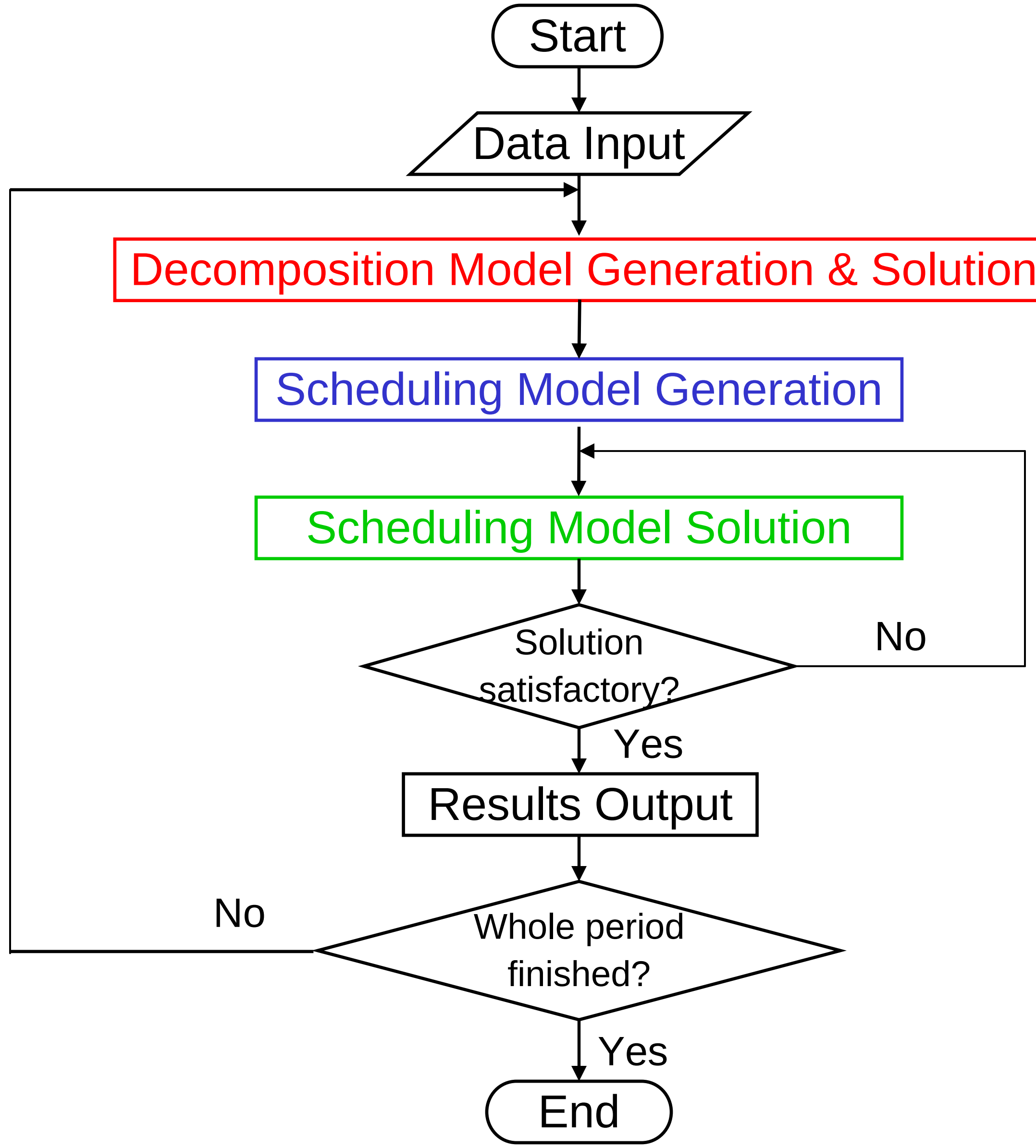
Model and Solution Statistics for Motivating Example

	Bounded Uncertainty in Processing Time (ε=0.15, δ=0.10)		Normal Uncertainty in Product Demand (ε=0.10, δ=0.10, κ=0.05)		Bounded and Symmetric Uncertainty in Market Prices (ε=0.05, δ=0.05, κ=0.05)	
	Nominal Solution	Robust Solution	Nominal Solution	Robust Solution	Nominal Solution	Robust Solution
Objective Function	3638.75	2887.19	8.007	8.222	1088.75	955.23
CPU Time (s)	0.44	18.35	0.02	0.02	0.02	0.02
Binary Variables	96	96	60	60	60	60
Continuous Variables	442	378	280	280	280	285
Constraints	553	713	375	377	334	345

Model and Solution Statistics for Motivating Example

	Poisson Uncertainty in Processing Time (ε=0.05, δ=0.20, κ=0.24)		Uniform Uncertainty in Product Demand (ε=0.10, δ=0.05, κ=0.00)		Normal Uncertainty in Market Prices (ε=0.05, δ=0.05, κ=0.05)	
	Nominal Solution	Robust Solution	Nominal Solution	Robust Solution	Nominal Solution	Robust Solution
Objective Function	3638.75	2887.19	8.007	8.174	1088.75	966.97
CPU Time (s)	0.46	11.33	0.02	0.02	0.02	0.05
Binary Variables	96	96	60	60	60	60
Continuous Variables	442	442	280	280	280	285
Constraints	553	777	375	409	334	334

Rolling Horizon Framework



Case Study 2: A Large-Scale Polymer Compounding Plant

Case Study 2: Plant Data Description

- Over 80 different products considered (250 overall)
- Basic operations: reaction, filtering, storing and filling
- Units: reactors, filter presses, prill tower, swing and product tanks, filling stations – (60+ units)
- Scheduling horizon: 18 days
- Storage limitations on reactors, tanks
- Given:
  - processing recipes for all products
  - unit suitability for each task of each product
  - capacity limits of each unit for a suitable task
  - processing time or processing rate of each task
  - clean-up time for each unit switching between tasks
  - inventory level of each product
  - demands: amount and due date

Case Study 2: Process Alternatives

State-Task Network (STN) representation

F → Reaction → I1 → Filter → I2 → Swing → P → Filling → P

F → Reaction → P → Filling → P

F → Reaction → I1 → Tank → P → Filling → P

F → Reaction → I1 → Swing → P → Filling → P

F → Mixed Prod → I1 → Tank → P → Filling → P

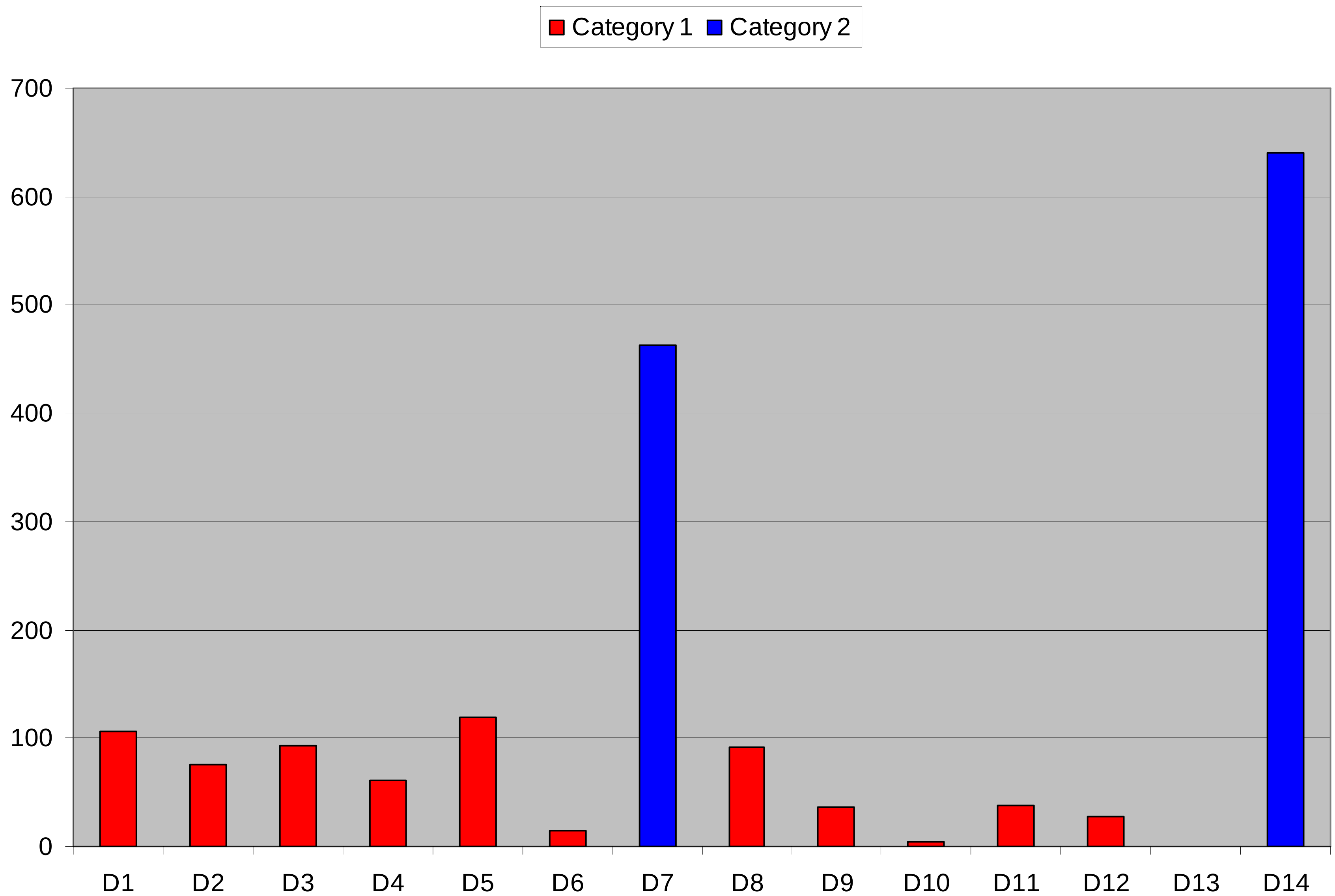
F → Reaction → M → Prill → I1 → Tank → P → Filling → P

F → Mixed Prod → I1 → Tank → I2 → Tank → P → Filling → P

Case Study 2: Challenges

- Inherent complexity of the physical problem
  - Long time horizon
  - Intermediate due dates and demands
  - Many products (250)
  - Many units (60)
  - Storage considerations
  - Clean-up – sequence dependent
- Special constraints and restrictions
  - Differentiate between category 1 and category 2 products
  - Campaign mode production (sequence-dependent) for prill tower
- Leads to large-scale combinatorial problem with many binary variables which must consider a tradeoff between quality of solutions and required computational resources

Case Study 2: Distribution of Demands

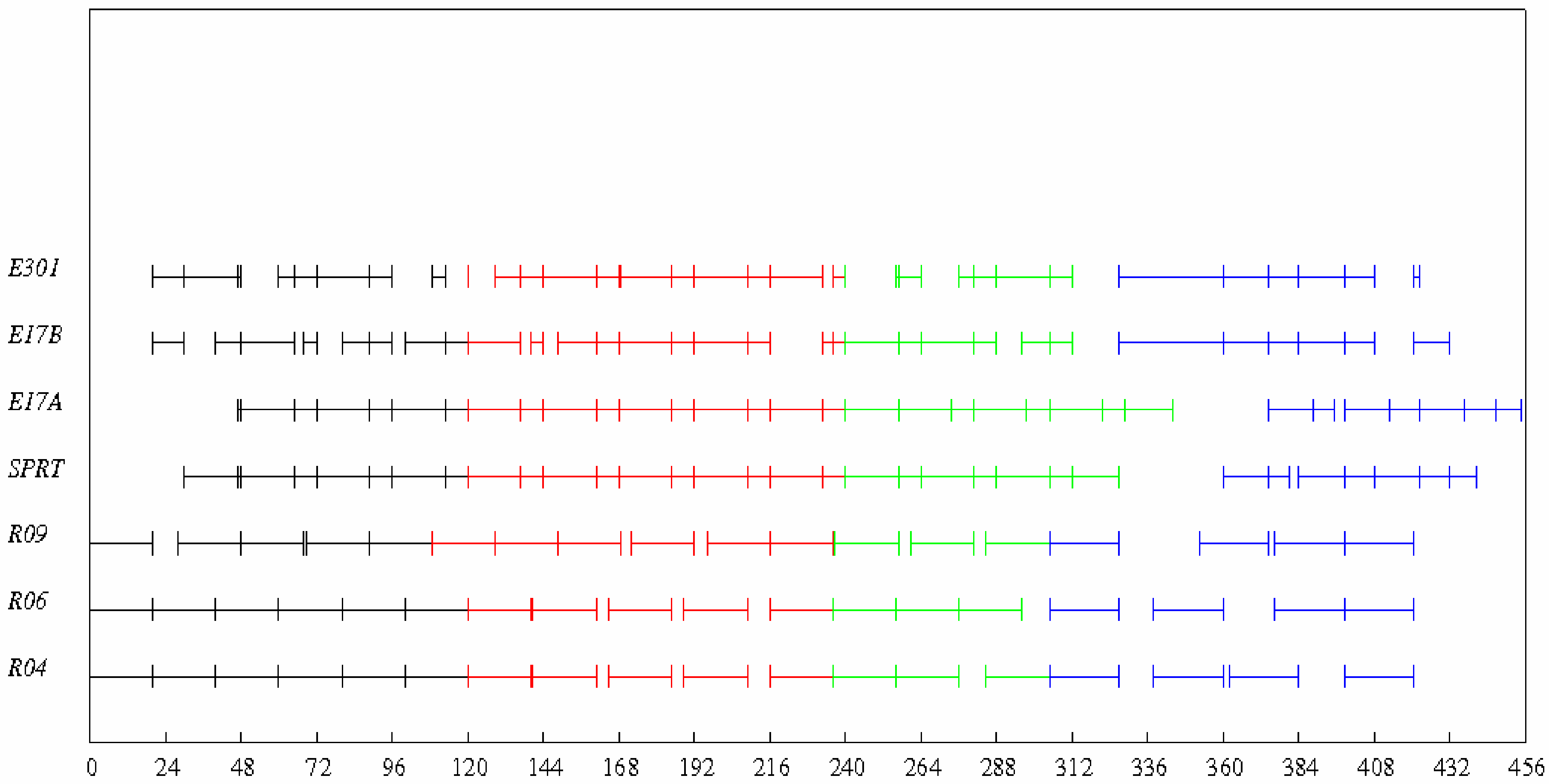


Case Study 2: Decomposition Results

- Production in 18 days (with campaign mode production) decomposed into eight time horizons

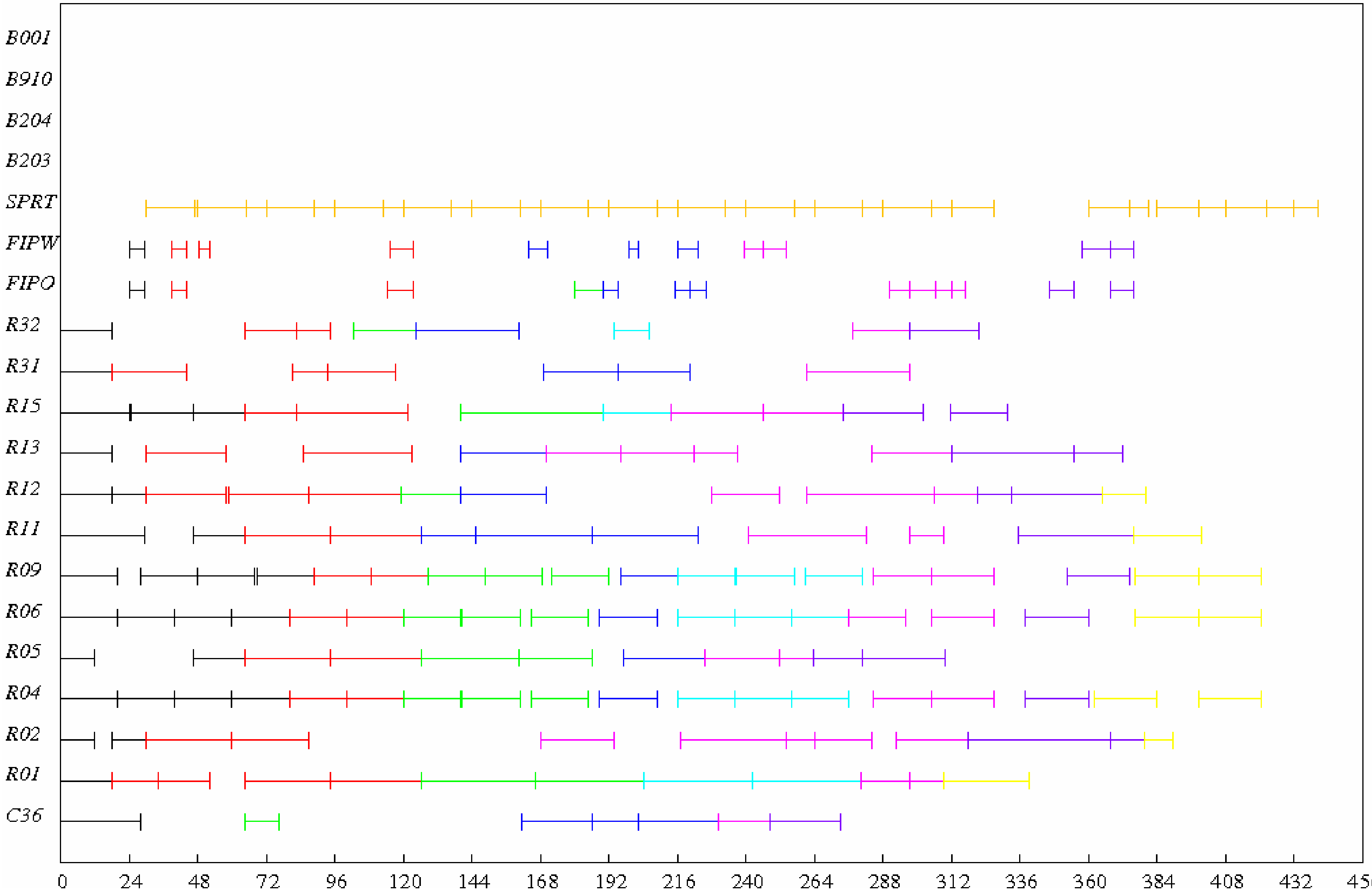
Time horizon	Days	Main products	Additional Products
1	0-2	27	1
2	3-4	31	0
3	5-6	51	0
4	7-8	53	0
5	9-10	43	0
6	11-12	53	0
7	13-14	56	0
8	15-18	40	0

Case Study 2: Campaign Mode Production



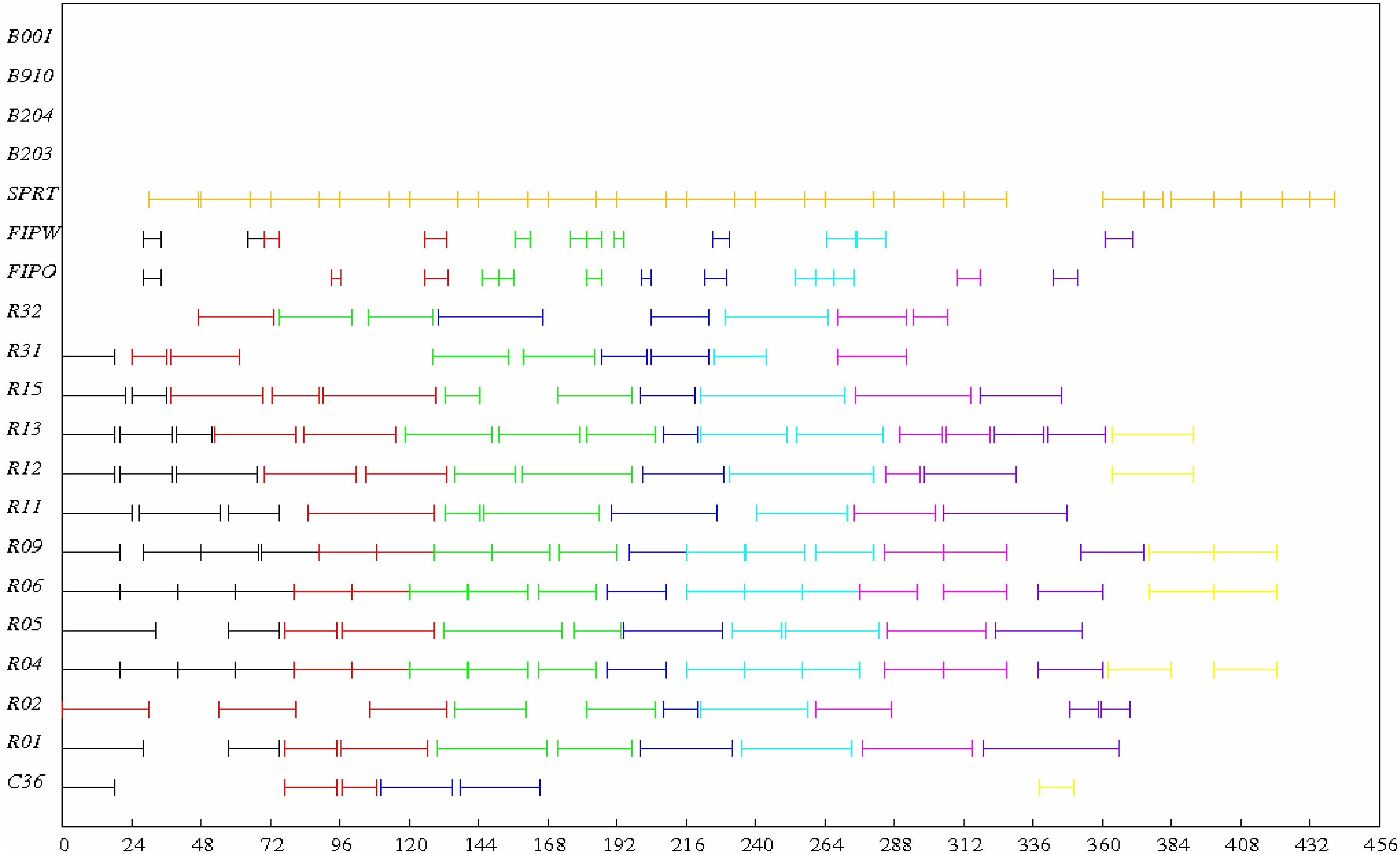
Nominal Schedule for Campaign Mode Production Units

Case Study 2: Nominal Solution



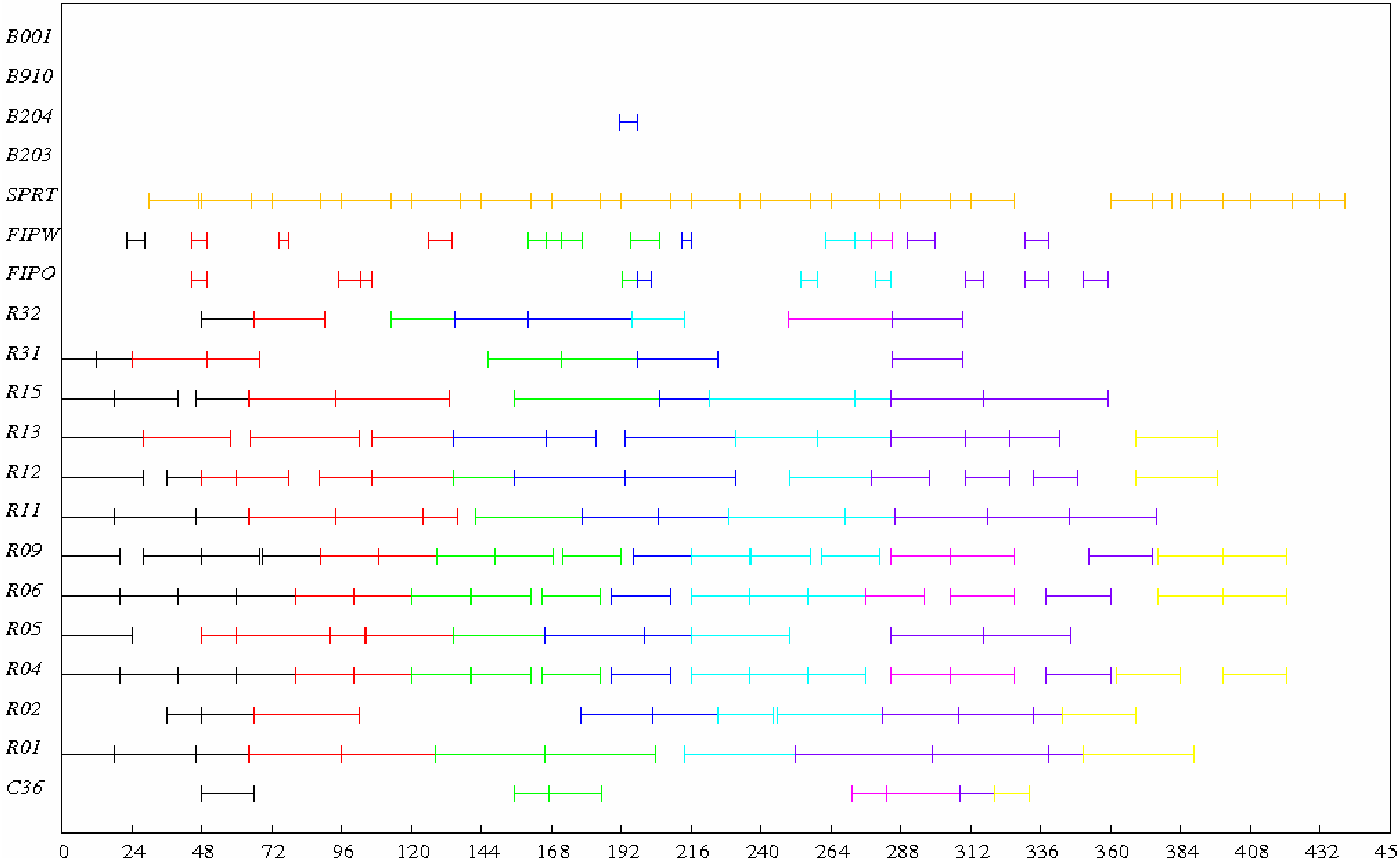
Nominal Schedule with Campaign Mode Production

Case Study 2: Robust Solution



- Robust Schedule with campaign mode production and bounded uncertainty (ε = 10%, δ = 0%) in fixed processing times of reactor tasks

Case Study 2: Robust Solution



- Robust Schedule with campaign mode production and bounded uncertainty (ε = 10%, δ = 0%) in intermediate demand of all products

Results with Campaign Mode Production for the First Horizon, H1

	Nominal	Robust: Time (ε=10%, δ=0%)	Robust: Demand (ε=10%, δ=0%)
Binary Variables	4914	4914	4914
Continuous Variables	34778	36114	34778
Constraints	172846	284734	172846
Objective Function	-18404.23	-16776.87	
No. of Uncertain Parameters	0	26	13

Scheduling under Uncertainty: Conclusions

- Uncertainty in production scheduling comes from model parameters and environmental data. Solutions obtained at nominal values can be unreliable.
- A novel Robust Optimization approach is proposed to generate "robust" solutions for MILP problems by transforming the stochastic problem into its deterministic counterpart. Different types of uncertainty addressed: bounded; bounded and symmetric; with known distribution, e.g. uniform, normal, difference of normal, discrete, binomial, poisson distributions.
- Robust Optimization is applied to scheduling problems with uncertain processing times, product demands, and market prices.
- Results show effectiveness, rendering the potential to solve large problems, and provide insights on tradeoffs between conflicting objectives.