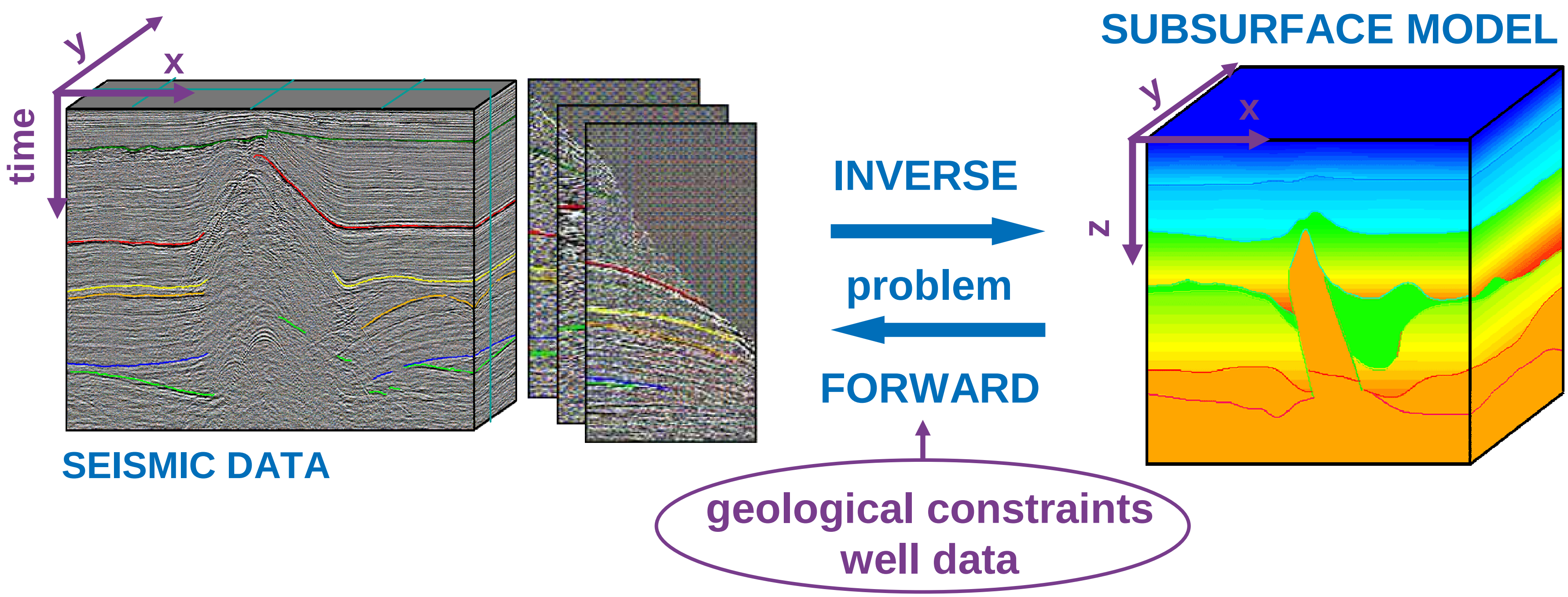
Optimization and uncertainty analysis for seismic inverse problems

Delphine Sinoquet, Frédéric Delbos, Carole Duffet

Seismic inverse problems

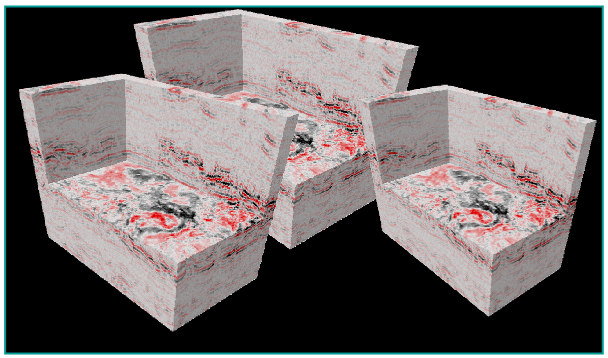
- **Seismic tomography : traveltime inversion for seismic velocity determination** **for seismic imaging of subsurface**



Forward problem : ray tracing (HF approx. of wave eq)

Seismic inverse problems

- **Seismic inversion for impedance determination**
  for reservoir delineation and characterization

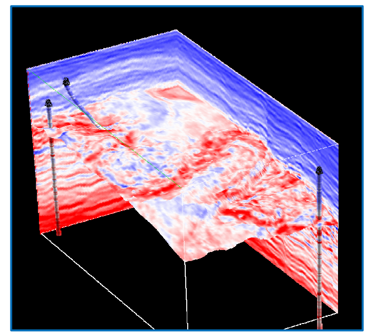


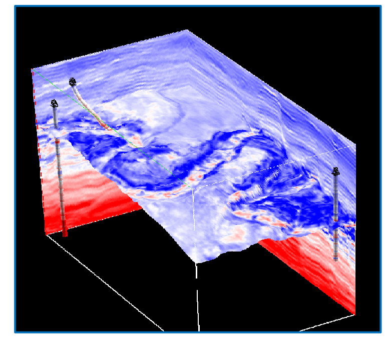
SEISMIC DATA

INVERSE

problem

FORWARD





IMPEDANCES

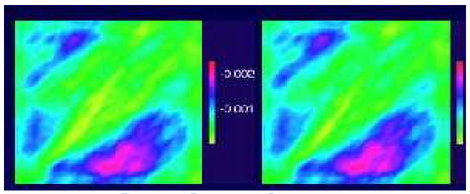
a priori geological model
built from well data, geological interpretation, velocity model

Forward problem : convolution of given wavelet and reflection coefficient

Seismic inverse problems

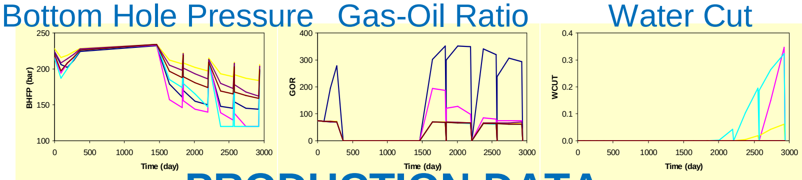
- **History matching of production data and 4D seismic data for characterization of dynamic behavior of reservoir during the production of a field**

Time 1 Time 2



4D SEISMIC DATA : impedance maps

Bottom Hole Pressure Gas-Oil Ratio Water Cut



PRODUCTION DATA

INVERSE

problem

FORWARD

- **Petrophysical parameters: Porosity and permeability Fault properties**
- **Well parameters: Skin, PI ...**

Forward problem : fluid flow simulation in reservoir
petro-elastic modelling

Seismic inverse problems

- **Traveltime inversion for seismic velocity determination**
  - **10^4 velocity parameters**
  - **Time consuming non-linear forward problem (1-2 hours)**
  - **constraints in optimization : a priori information, well data ...**

- **Seismic inversion for impedance determination**
  - **10^9 parameters**
  - **simplified forward problem = weakly non-linear**

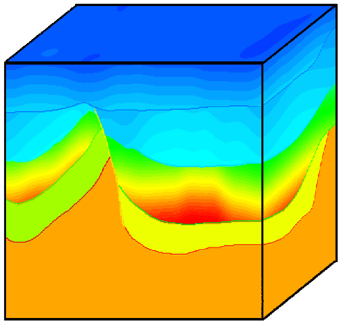
- **History matching of production data and 4D seismic**
  - **~100 parameters**
  - **Time consuming forward problem (several hours)**
  - **gradients are usually not available**

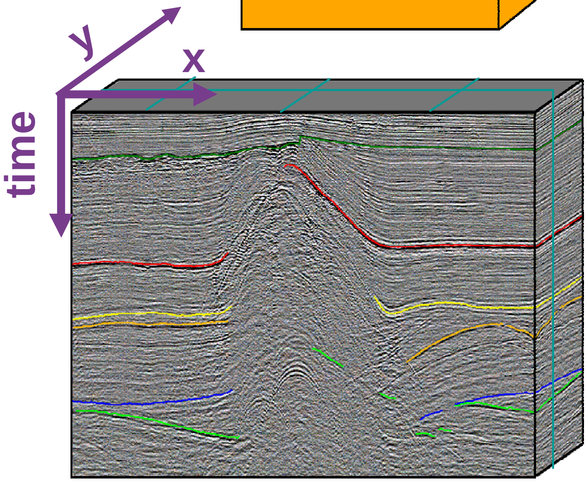
Seismic tomography

- **Model composed of**
  - **layer interfaces**
  - **velocity variations within layers**

    both modeled by B-spline functions
- **Data : interpreted traveltimes from seismic data**

  recognize traveltimes associated with reflections on a geological layer interface
- **Least-square formulation**





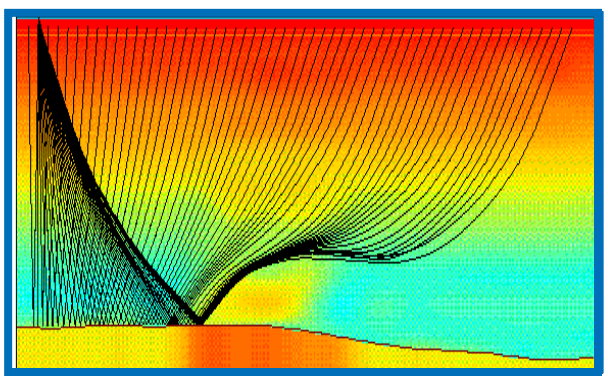
$$\min_m \left(\|T(m) - T^{obs}\|^2 + \|m - m^{apriori}\|^2_{C_m} \right)$$

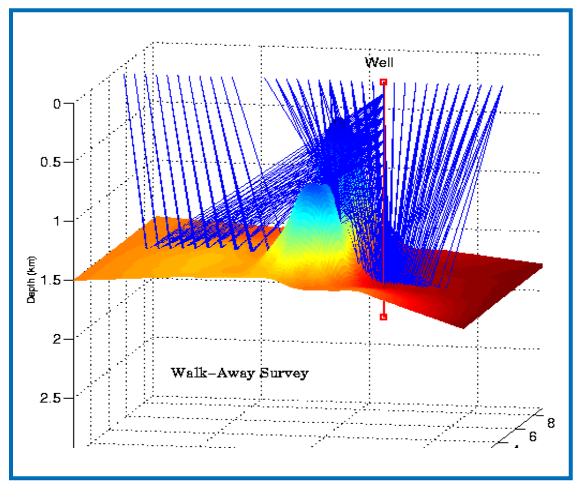
Seismic tomography

- **Forward problem : ray tracing**

$$m \in \mathbb{R}^{n_p} \longrightarrow T(m) \in \mathbb{R}^{n_d}$$

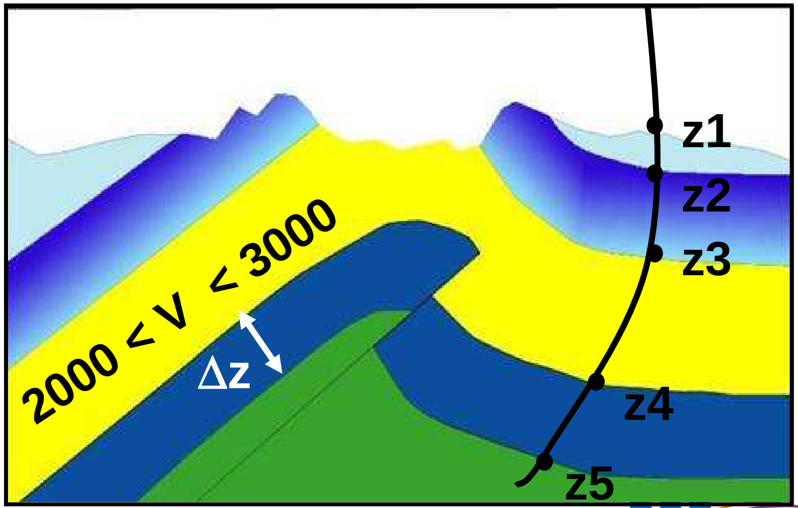
  - **CPU time consuming : lot of (source, receiver) couples**
  - **non-linear operator : complex wave propagation in the subsurface**
  - **cheap computation of the Jacobian matrix**





Seismic tomography

- **Constrained non-linear optimization**
  - **Large number of constraints**
  - **Large variety of constraints**
    - **of different physical natures: on velocity variations, on interface depths, on their derivatives (e.g. slope of an interface, velocity gradient …)**
    - **equality and inequality**
    - **local or global constraints**



Non-linear constrained optimization

$$\min_m \left(\|T(m) - T^{obs}\|^2_{C_d} + \|m - m^{apriori}\|^2_{C_m} \right)$$
$$Em = e$$
$$l \leq Cm \leq u$$

- **~1000 of linear constraints**
- **Difficulty : determining which inequality constraints are active (among 3^{ni} possibilities)**

➢ ***a dedicated non-linear constrained optimization method***

Non-linear constrained optimization

2 main approaches are possible:

- **Penalty methods:**
  minimization of a sequence of non-linear functions "cost + constraints" (e.g. Interior Points)

- **SQP methods (Sequential Quadratic Programming):**
  minimization of a sequence of quadratic problems subject to constraints

Non-linear constrained optimization

2 main approaches are possible:

- **Penalty methods:**
  minimization of a sequence of non-linear functions "cost + constraints" (e.g. Interior Points)
  - ***more non-linear function evaluations***

- **SQP methods (Sequential Quadratic Programming):**
  minimization of a sequence of quadratic problems subject to constraints
  - ***each iteration of SQP is complex***

Non-linear constrained optimization

- **A Gauss-Newton SQP approach**

  at iteration *k*, solve a quadratic pb under linear constraints

$$\min_{\delta m}\left(F_k(\delta m) := \|J_k(\delta m) + T(m_k) - T^{obs}\|^2_{C_d} + \|\delta m + m_k - m^{apriori}\|^2_{C_m}\right)$$
$$E\delta m = \tilde{e_k}$$
$$\tilde{l_k} \le C\delta m \le \tilde{u_k}$$

- **Augmented Lagrangian method: well-adapted method to solve large optimization problem**

  solve a sequence of quadratic pb subject to BOUND constraints

$$\min_{\delta m, y}\Big(L_k(\delta m, y) := F_k(\delta m) \quad +{\lambda_E^l}^T(E\delta m - \tilde{e_k}) + r\|E\delta m - \tilde{e_k}\|^2$$
$$+{\lambda_I^l}^T(C\delta m - y) + r\|C\delta m - y\|^2\Big)$$
$$y = C\delta m \qquad (y \text{ auxilliary variable})$$
$$\tilde{l_k} \le y \le \tilde{u_k}$$

Non-linear constrained optimization

- **Gauss-Newton SQP method:**
  sequence of quadratic problems subject to constraints

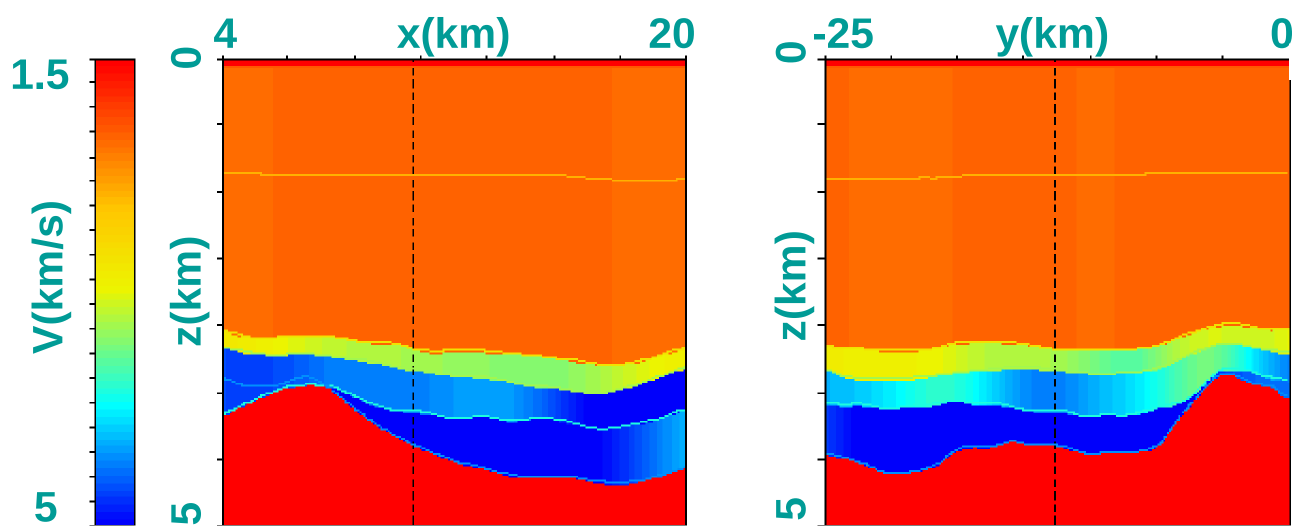
  - **Augmented Lagrangian method:**
    sequence of quadratic problems subject to BOUND constraints

    - **determination of the active bound constraints via an active set method**
    - **minimization of the quadratic function on the determined active set via a preconditioned conjugate gradient**

An application of reflection tomography

- **Application on a 3D North Sea dataset***
  - **layer-stripping approach**
  - **strong under-determination in Tertiary layer**
  - **127569 traveltime data**
  - **5960 unknowns**

Unconstrained optimization result



RMS of traveltime misfits = 6.1ms

***courtesy of bp**

An application of reflection tomography

- **Proposal:**
  - **introduction of 2300 constraints**
  - **global inversion to avoid bad data fitting for deep layers often observed with the layer stripping approach**

Constraints		Model obtained with the UNCONSTRAINED inversion	Model obtained with the CONSTRAINED inversion
Mean depth mismatch at the 5 well locations	tpal	96m	0m
	tchalk	132m	0m
	bchalk	140m	0m
Vertical velocity gradient in Tertiary	0.1<k<0.3/s	k=0/s	k~0.18/s
Velocity range	2.5 < vpal < 4km/s	ok	ok
	3.5 < vichalk < 5.7km/s	ok	ok
	4.2 < vchalk < 5.8km/s	ok	ok

+ constraints on layer thickness'

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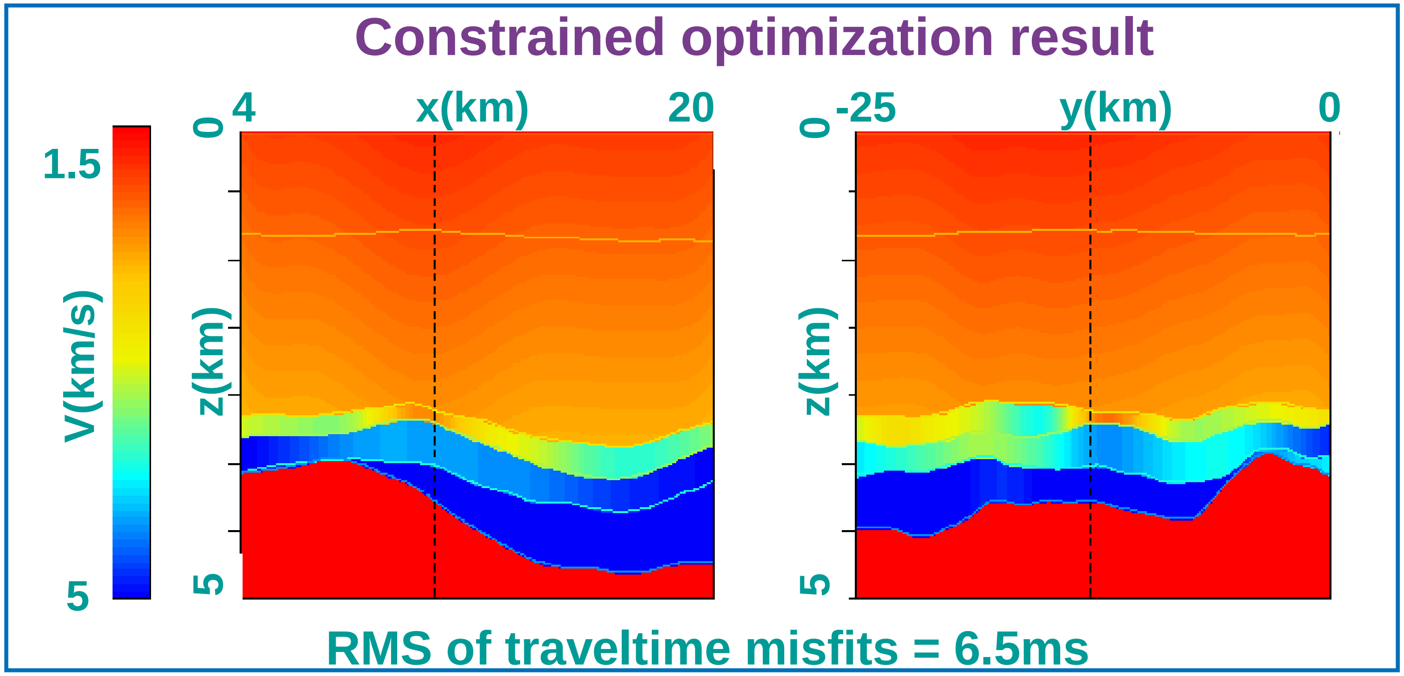
An application of reflection tomography

- **Solution model of constrained optimization:**



An application of reflection tomography

- **6 Gauss-Newton iterations (9 function evaluations)**
- $$\frac{\textbf{CPU time (1 iteration with constraints)}}{\textbf{CPU time (1 iteration without constraint)}} = \mathbf{3.19}$$

- **a few number of Gauss-Newton iterations is required**
- **the chosen activation method is efficient even for a large number of constraints**
- **no additional weight to be tuned (automatic tuning of the augmentation parameter)**

Uncertainty analysis

- **Linearized framework: analysis of the a posteriori covariance matrix around the solution**

$$C'_m = (J^T C_d J + C_m^{-1})^{-1}$$

  - $\left(\sqrt{C'_m}\right)_{ii}$ **uncertainties on the inverted parameters**
  - $\left(C'_m\right)_{ij}$ **correlation between the uncertainties**

C'_m **may be a huge dense matrix: expensive computation for 3D pb**

- **Our methods to estimate the uncertainties**
  - **Simulations of admissible models**
  - **Uncertainties on geological macro-parameters**
  - **Non linear approach : exploration of admissible space thanks to constrained optimization**

Simulation of admissible models

- **Sample the a posteriori probability density function**

$$\exp\left(-\frac{1}{2}\delta m^T C_m'^{-1}\delta m\right) = \exp\left(-\frac{1}{2}\delta m^T U^T U \delta m\right)$$

via a Choleski decomposition of the Hessian $H = C_m'^{-1}$

sample the gaussian probability density function with unit variance via the variable transformation $\delta m' = U\delta m$

- **We have access to a range of likely models**
- **But for 3D problems:**
  Choleski decomposition may be too expensive

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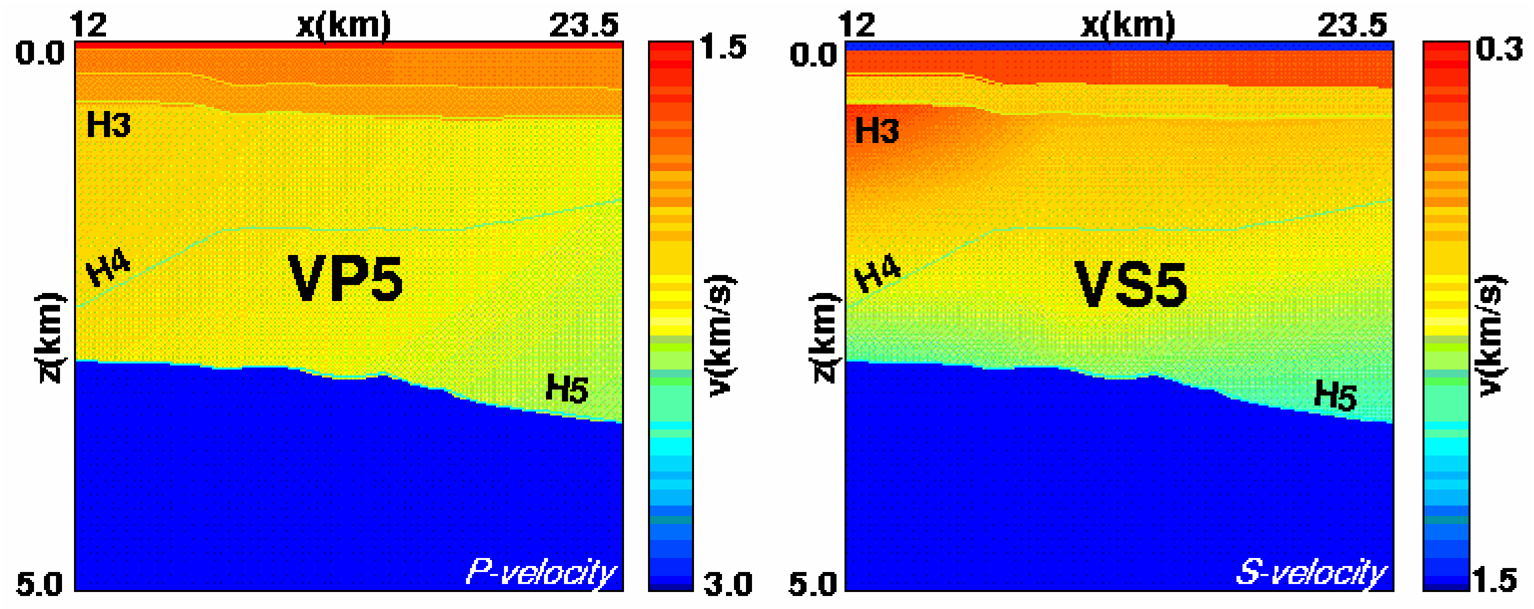
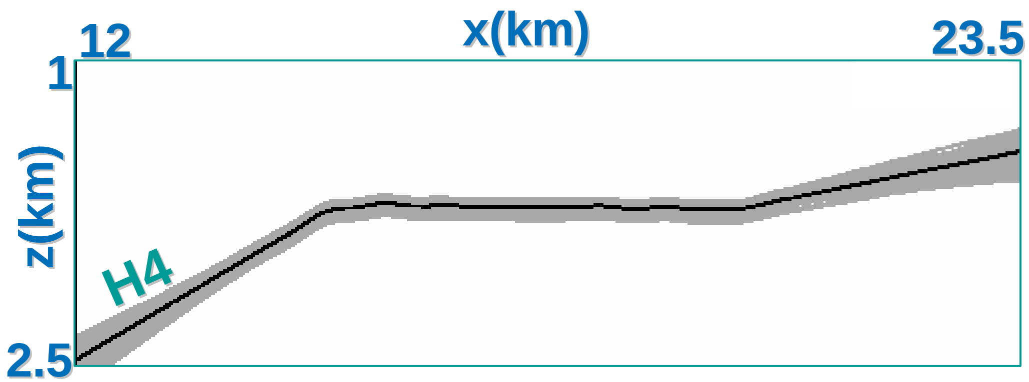
Solution model of inversion

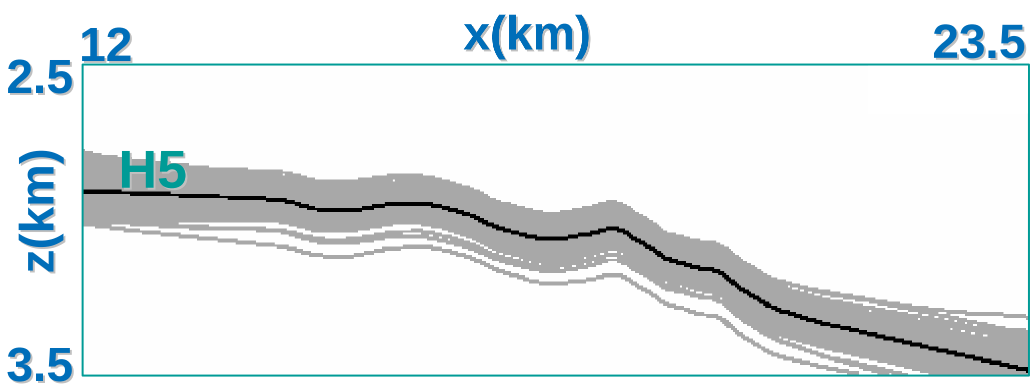


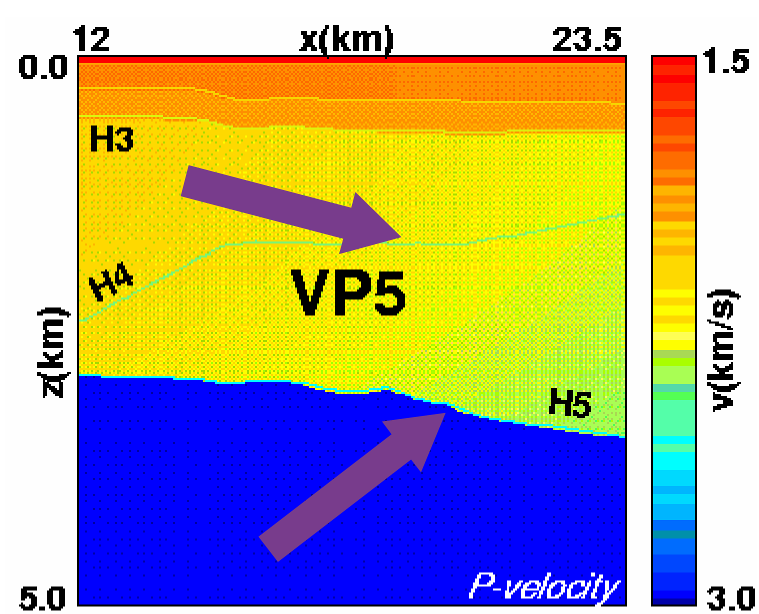
RMS traveltime misfits = 6.2 ms

η = 6.29 % and δ = - 4.43 %

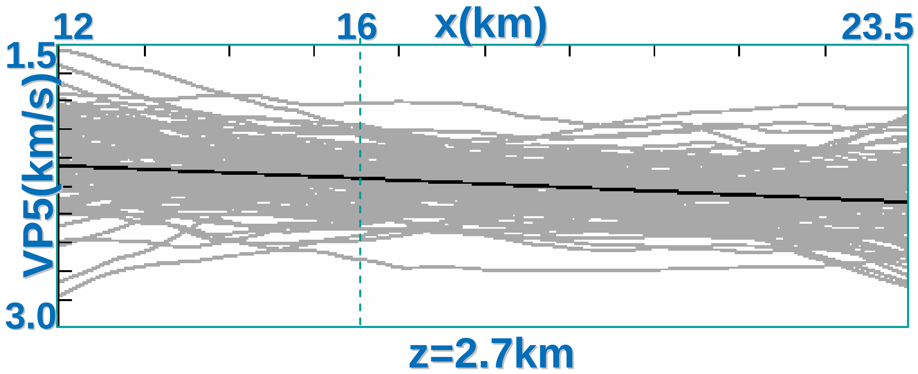
Simulation of admissible models

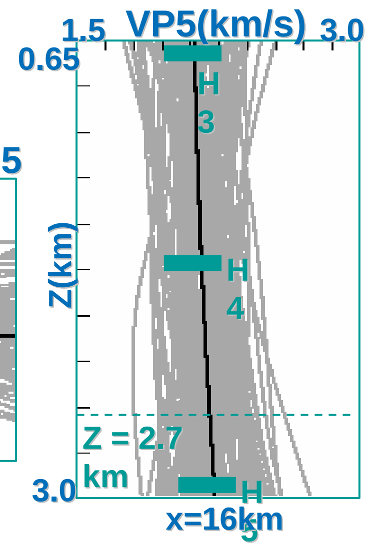


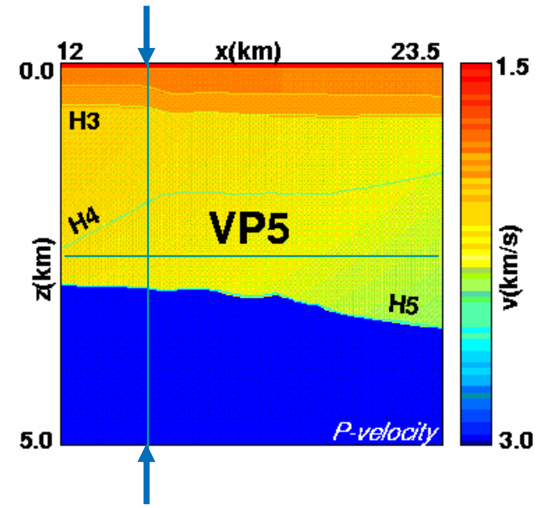


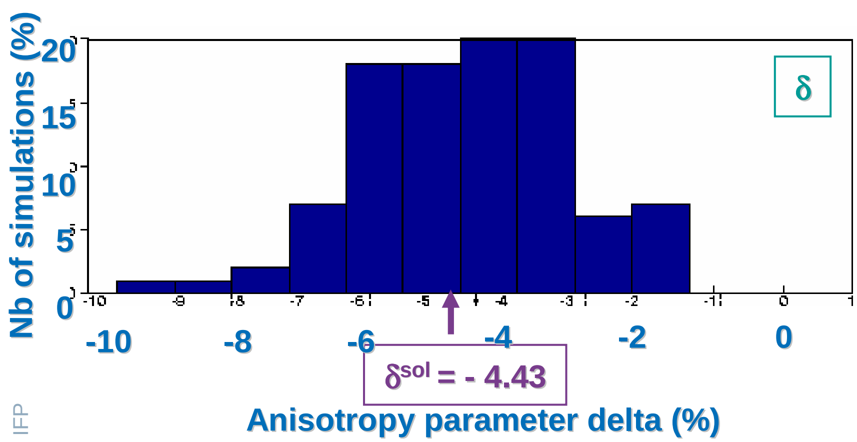


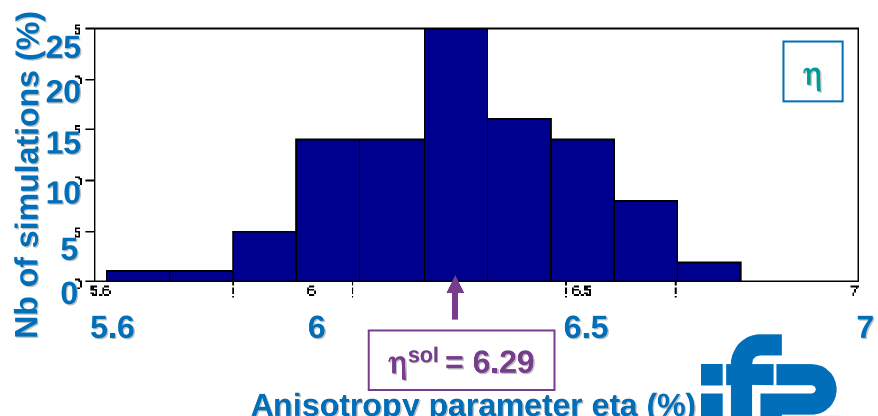
Simulation of admissible models











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Macro-parameters

- **limit the uncertainty analysis to quantities which are interesting for the geophysicist**
  - **slope of an interface**
  - **vertical variations of the velocity in a region**
  - **layer thickness**
  - **…**
- **define a macro-parameter:** $P = Bp$
  a linear combination of parameters
- **Reduced a posteriori covariance matrix in the macro-parameter space**

Macro-parameters

	VP5	VS5	η	δ	H4	H5
VP5	475.1 m/s	0.002	-0.03	-0.02	0.005	0.01
VS5	0.002	168.9 m/s	-0.04	-0.03	0.005	0.01
η	-0.03	-0.04	0.22%	0.93	-0.16	-0.33
δ	-0.02	-0.03	0.93	1.6%	-0.17	-0.36
H4	0.005	0.005	-0.16	-0.17	77.1 m	0.06
H5	0.01	0.01	-0.33	-0.36	0.06	80.3 m

Macro-parameter = mean of the depth of the interface
Macro-parameter = mean of the velocity

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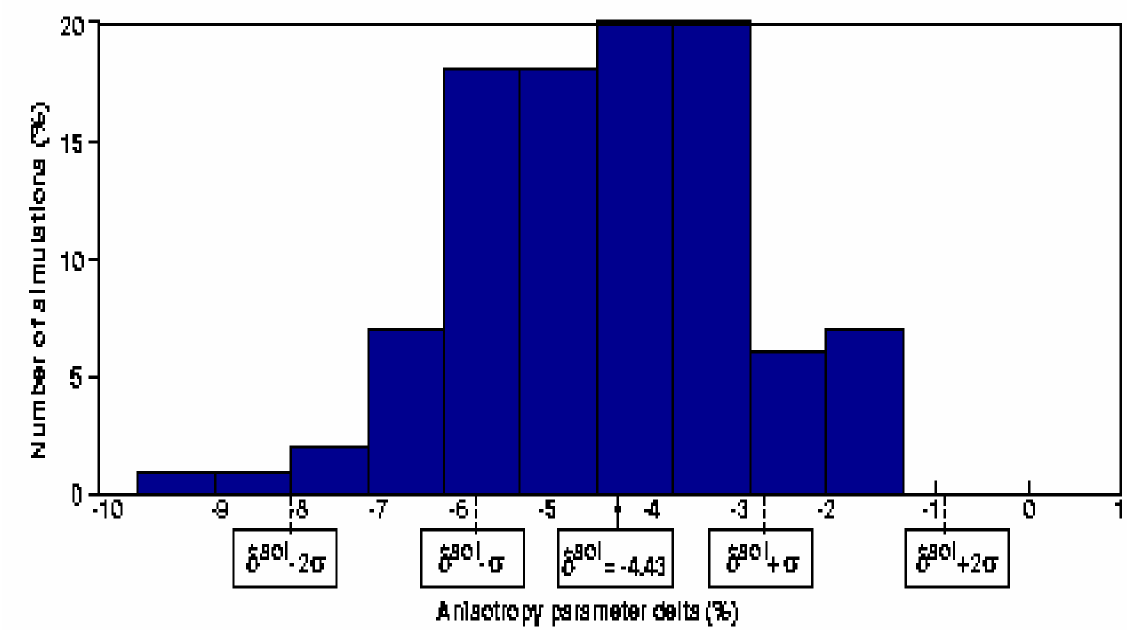
Non linear a posteriori analysis

- **Motivations: limitations of the linearized approach**
  - **limitations of the quadratic approximation of the non linear cost function**

- **Test other geological scenarii**
  - **try to delimit the space of admissible solutions**
  - **an experimental approach: solve the inverse problem under geological constraints**

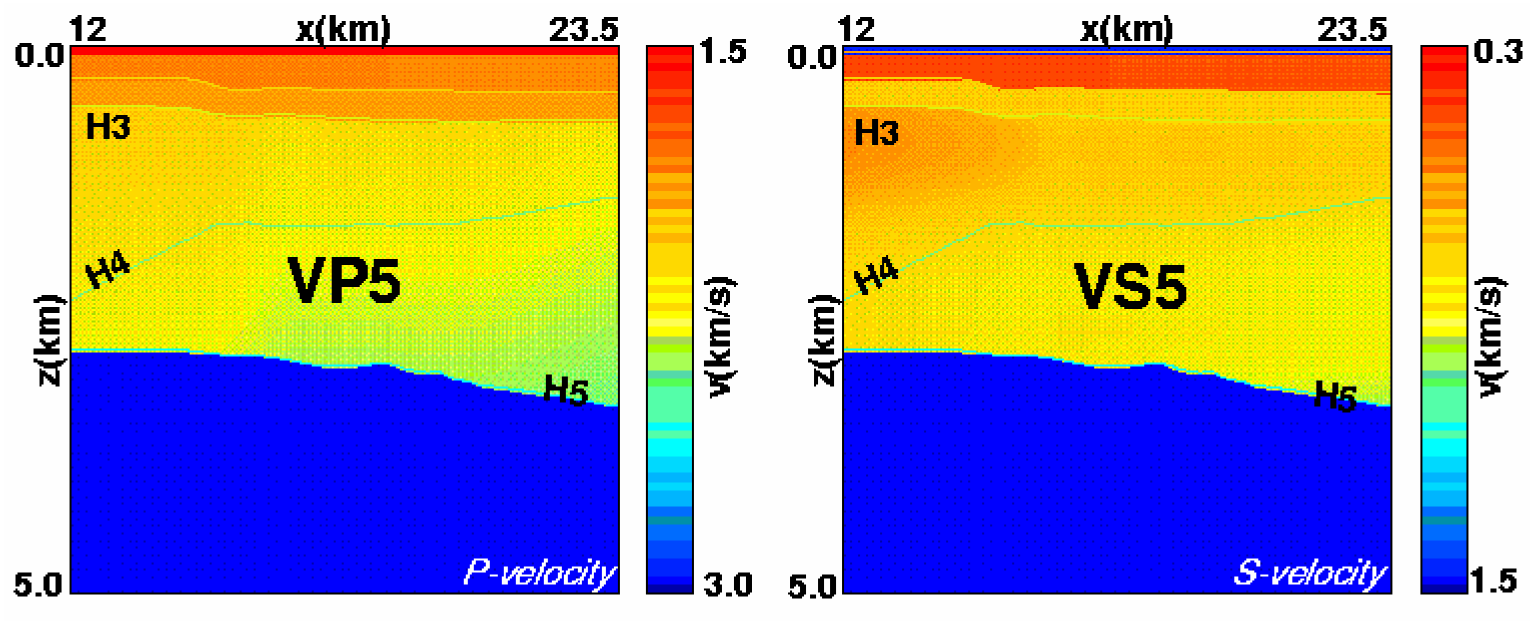
Non linear a posteriori analysis

- **Test other geological scenarii**
  - **test an hypothesis on δ values :**
    could we find a model that fits the data with δ > 0 ?
    whereas simulation approach furnishes only models with δ < 0



An other solution

Constraint = $\delta > 0$



RMS traveltime misfits = 6.4 ms

η = 6.2 % and δ = 2 %

Conclusions

- **Optimization**
  - **Develop a dedicated optimization method to handle constraints**
  - **Allow to integrate lot of different types of additional data geological data, well data …**

- **Uncertainty analysis**
  - **Linearized approach : Hessian matrix**
  - **Non-linear approach : guided by geological constraints**

References

Seismic tomography

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Seismic impedance inversion

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History matching

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delphine.sinoquet@ifp.fr

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