Verification and Validation in Ocean Modeling

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Mathematical Methods for V&V

Sandia, August 14-16, 2007

Collaborators

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- Paul Fischer, Argonne National Laboratory
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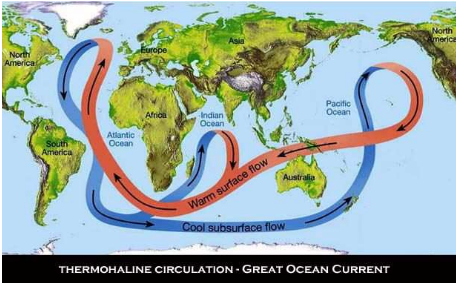
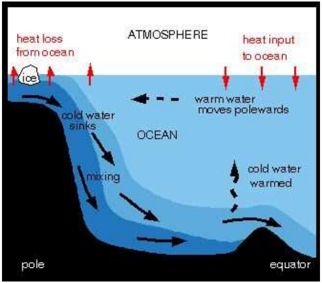
Thermohaline Circulation





Gravity Currents

- density currents, overflows
- flow of a fluid within another fluid
- driven by the gravitational force
- Kelvin-Helmholtz rolls

Motivation

Protection against the bore in the Qiantang river, 910 AD.

Motivation

- importance
  - oceanic gravity currents – thermohaline circulation
  - polar seas – cooling
  - marginal seas – evaporation
- challenges
  - climate models: $Earth - \mathcal{O}(100\ km)$
  - basin models: $\mathcal{O}(1000\ km) - \mathcal{O}(10\ km)$
  - regional/coastal models: $\mathcal{O}(100\ km) - \mathcal{O}(1\ km)$

Motivation

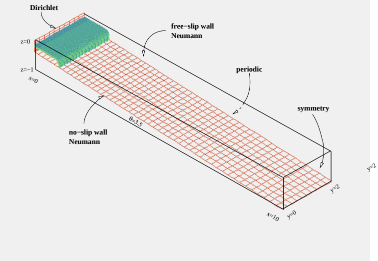
- laboratory tank experiments
  - continental slope – $\mathcal{O}(1^\circ)$
  - laboratory tank – $\mathcal{O}(10^\circ)$
  - ocean - stratified
  - laboratory tank - stratification difficult

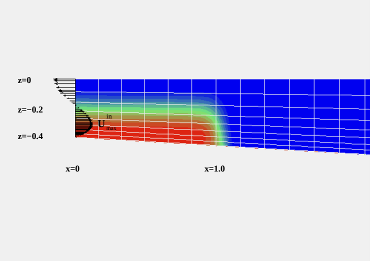
Boussinesq Equations

$$\begin{aligned} \mathbf{u}_t - \triangle \mathbf{u} + (\mathbf{u} \cdot \nabla)\, \mathbf{u} + \nabla p - Ra\, T\, \mathbf{k} &= \mathbf{f} && \text{in } \Omega \\ \nabla \cdot \mathbf{u} &= 0 && \text{in } \Omega \\ T_t - Pr^{-1} \triangle T + (\mathbf{u} \cdot \nabla)\, T &= 0 && \text{in } \Omega \end{aligned}$$

$$\begin{aligned} \text{Rayleigh number} \quad Ra &:= \frac{g\, \beta\, \triangle T\, H^3}{\nu^2} \\ \text{Prandtl number} \quad Pr &:= \frac{\nu}{k} \end{aligned}$$

Configuration & Parameters



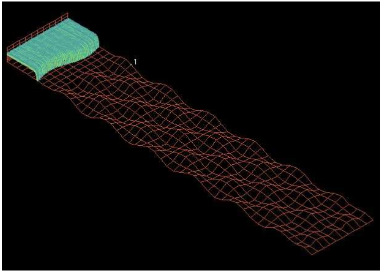


$$Ra = 5 \times 10^6, \quad Pr = 1$$

2D Smooth Topography

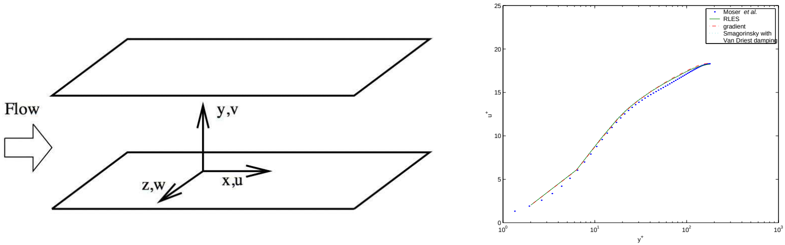
Özgökmen, Fischer, Duan, Iliescu,
Journal of Physical Oceanography, 2004

3D Complex Topography



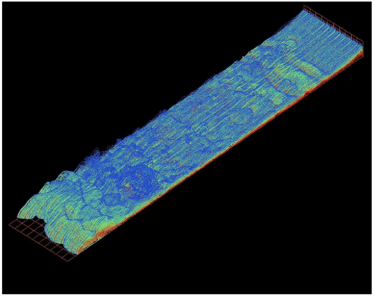
Özgökmen, Fischer, Duan, Iliescu,
Geophysical Research Letters, 2004

Validation & Verification



Iliescu, Fischer,
Physics of Fluids, 2003

Validation

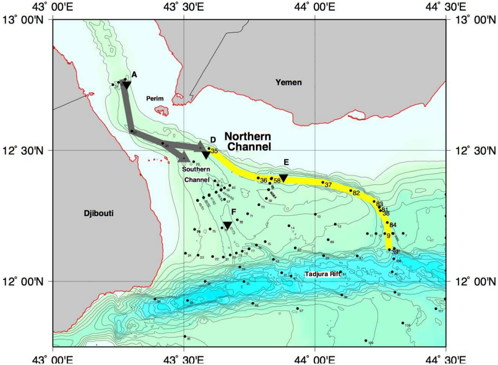
 

Laboratory experiments (left), numerical results (right).

Validation & Verification

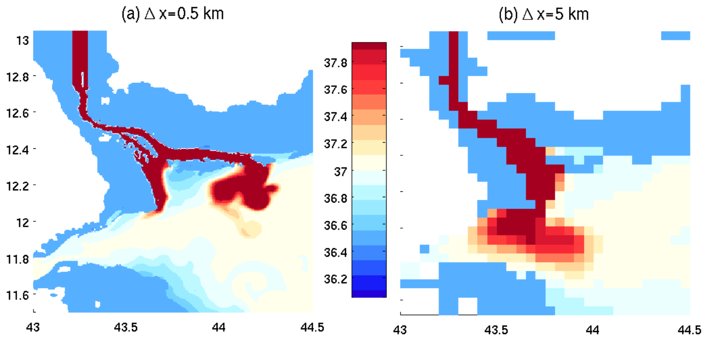
- Compared
  - 2D numerical results
  - 3D numerical results
  - laboratory experiments
- Criteria
  - propagation speed
  - growth rate of gravity current head
  - entrainment
- Conclusions
  - 2D, 3D – different
  - 3D, laboratory experiments – similar

Red Sea



REDSOX winter cruise: topography and plume survey stations.

Red Sea



Red Sea overflow with HYCOM.

Computational Cost

- *all* scales
- $N \sim \mathcal{O}(Re^{9/4})$
- $U \sim 1\, m/s, \;\; L \sim 100\, m \Longrightarrow Re_{GC} \sim 10^8$
- $N_{GC} \sim \mathcal{O}(Re_{GC}^{9/4}) \sim 10^{18}$
- alternatives needed

Large Eddy Simulation

- approximate *large* scales
- model *small* scales
- spatial filter g_δ
- $\overline{\mathbf{u}} := g_\delta * \mathbf{u}$
- $g_\delta *$ NSE

Navier-Stokes

- closure problem: *model* $\tau := \overline{\mathbf{u}\mathbf{u}} - \overline{\mathbf{u}}\,\overline{\mathbf{u}}$
- Smagorinsky

$$\tau = -(C_S\,\delta)^2\,\|\mathbb{D}(\overline{\mathbf{u}})\|_F\,\mathbb{D}(\overline{\mathbf{u}})$$

  - $\mathbb{D}(\overline{\mathbf{u}}) = (\nabla\overline{\mathbf{u}} + \nabla\overline{\mathbf{u}}^T)/2$ deformation tensor
  - $C_S \sim 0.17$ Smagorinsky constant
  - δ filter radius
- energy cascade $\Longrightarrow$ eddy-viscosity LES models

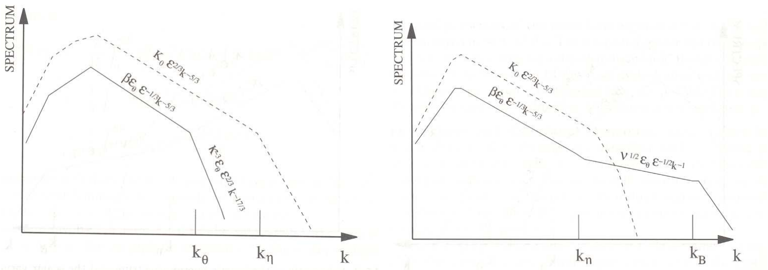
Boussinesq

- closure problem: *model* $\sigma := \overline{\mathbf{u}T} - \overline{\mathbf{u}}\,\overline{T}$
- Smagorinsky

$$\sigma = -(\widetilde{C}_S\,\delta)^2\,\|\mathbb{D}(\overline{\mathbf{u}})\|_F\,\mathbb{D}(\overline{T})$$

  - $\mathbb{D}(\overline{\mathbf{u}}) = (\nabla\overline{\mathbf{u}} + \nabla\overline{\mathbf{u}}^T)/2$ deformation tensor
  - $\widetilde{C}_S \sim$?
  - δ filter radius
- energy cascade ????

Energy Cascade



Low *Pr* (left), large *Pr* (right).

Stratification

- Richardson number
- $Ri = \frac{\frac{g}{T_0} \frac{\partial T}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}$
- Ri = buoyancy / vertical shear
- Ri-dependent LES models
- anisotropy

LES models

- LES model A: classic Smagorinsky
- LES model B: *Ri*-dependent vertical diffusivity
- LES model C: *Ri*-dependent vertical viscosity
- LES model D: *Ri*-dependent vertical diffusivity and viscosity

Boussinesq Equations

$$
\begin{aligned}
\mathbf{u}_t - Re^{-1}\,\triangle\mathbf{u} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} + \nabla p + Fr^{-2}\,\rho'\,\mathbf{k} &= \mathbf{f} && \text{in } \Omega \\
\nabla\cdot\mathbf{u} &= 0 && \text{in } \Omega \\
\rho'_t - Re^{-1}\,Pr^{-1}\triangle\rho' + (\mathbf{u}\cdot\nabla)\rho' &= 0 && \text{in } \Omega
\end{aligned}
$$

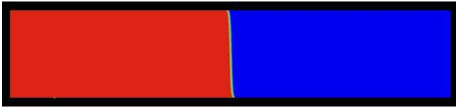
Reynolds number $Re := \dfrac{U\,L}{\nu}$

Froude number $Fr := \dfrac{U}{\sqrt{\dfrac{g\,\triangle\rho'\,L}{\rho_0}}}$

Prandtl number $Pr := \dfrac{\nu}{k}$

Dam Break

- computational domain

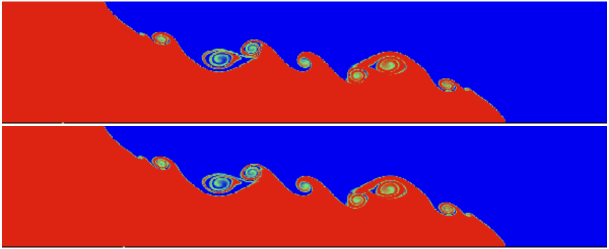


- boundary conditions
  - $\frac{\partial u}{\partial \mathbf{n}} = 0; \quad \frac{\partial w}{\partial \mathbf{n}} = 0; \quad (u, v, w) \cdot \mathbf{n} = 0$
  - $\frac{\partial \rho'}{\partial \mathbf{n}} = 0$
- spatial resolution: $N = 7500 \cdots 10^6$

DNS

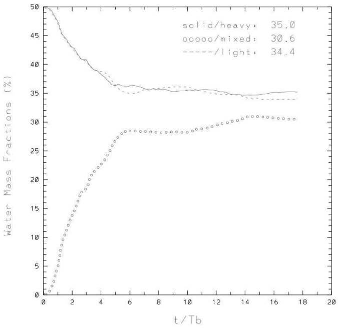
- $Re = 2800 \qquad N = 2.7 \times 10^5$
- $Re = 4300 \qquad N = 10^6$
- $Pr = 7$

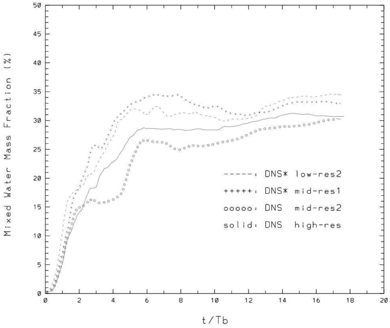
Verification



$Re = 4300,\ N = 2.7 \times 10^5$ (top), $N = 10^6$ (bottom).

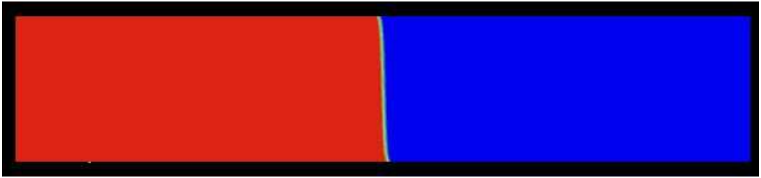
Criterion





$Re = 2800$, time evolution of water mass fractions (left), different resolutions (right).

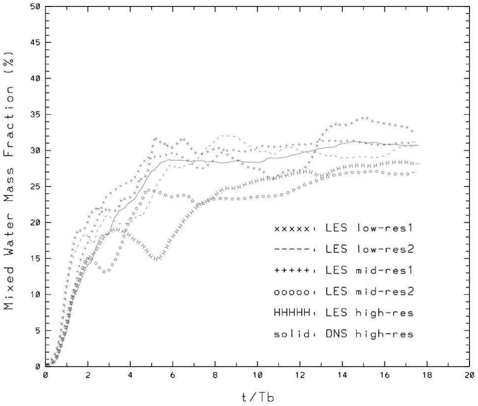
DNS



Özgökmen, Iliescu, Fischer, Srinivasan, Duan
Ocean Modelling, 2006

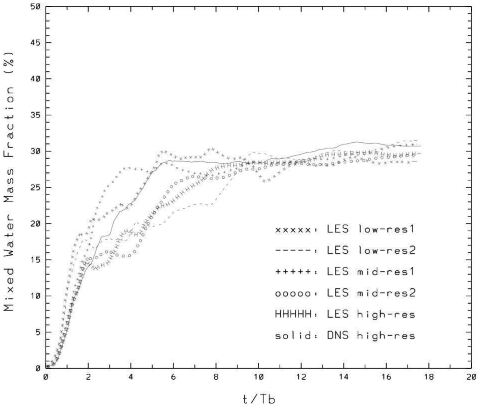
$Re = 4300, Pr = 7, N = 10^6.$

LES



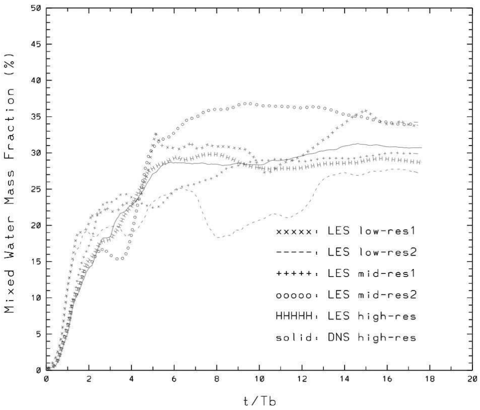
$Re = 2800$, time evolution of water mass fractions, LES model A.

LES



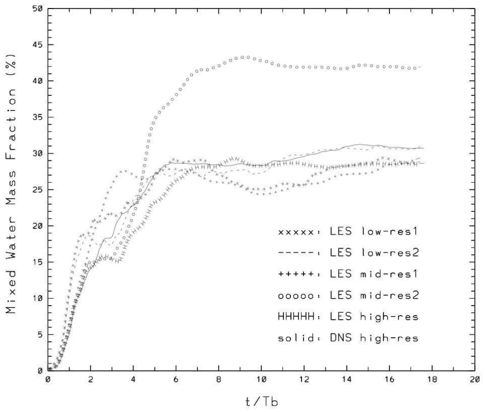
$Re = 2800$, time evolution of water mass fractions, LES model B.

LES



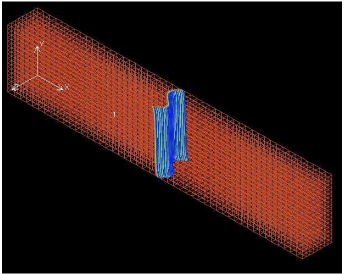
$Re = 2800$, time evolution of water mass fractions, LES model C.

LES



$Re = 2800$, time evolution of water mass fractions, LES model D.

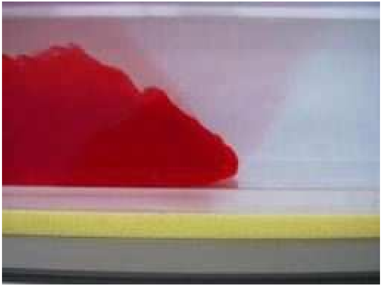
LES



Özgökmen, Iliescu, Fischer, Srinivasan, Duan
Ocean Modelling, 2006

$$Re = 2800, Pr = 7, N = 1.3 \times 10^6.$$

Validation



http://www.meteo.physik.uni-muenchen.de/řobert/meteorologielabor/labor.html

Supercomputer

Virginia Tech's System X.

Modeling

- Multidomain/Multiresolution Modeling
- Reduced-Order Modeling
- Stochastic Modeling

Funding

- **National Science Foundation** DMS-0209309

  3D Numerical Investigation of Density Currents

  2002 - 2006 $94,829.

- **National Science Foundation** OCE-0620464

  A New Modeling Framework for Nonhydrostatic Simulations of Small-Scale Oceanic Processes

  2006 - 2009 $147,861.