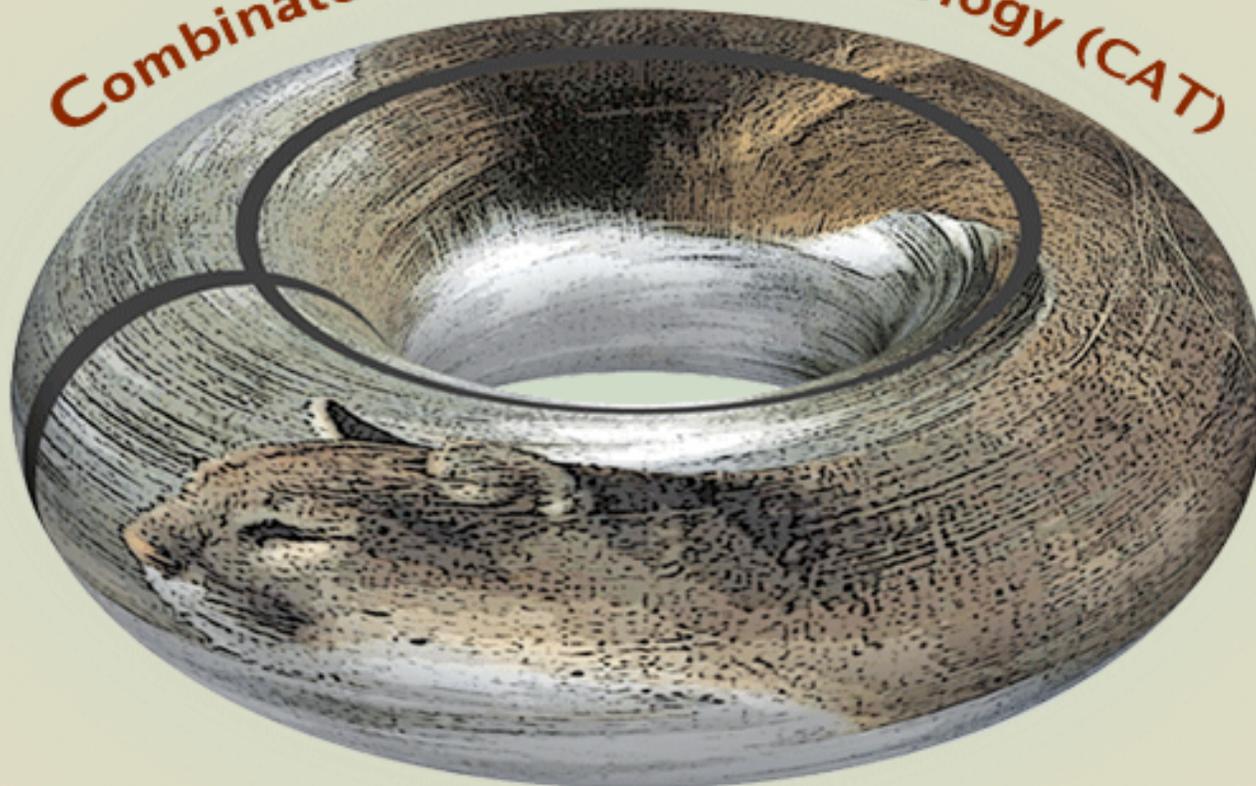


# Combinatorial Algebraic Topology (CAT)

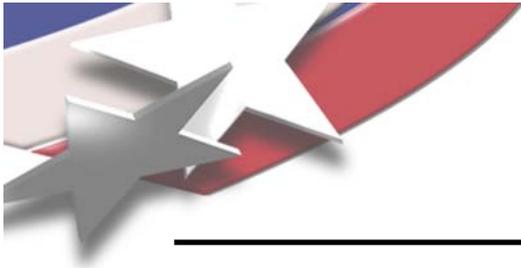


Welcome Address – Scott A. Mitchell

**Sandia National Laboratories - CSRI Workshop on  
Combinatorial Algebraic Topology (CAT): software, applications & algorithms**

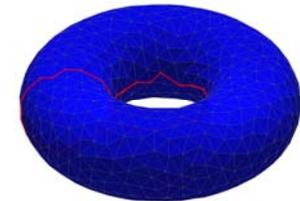
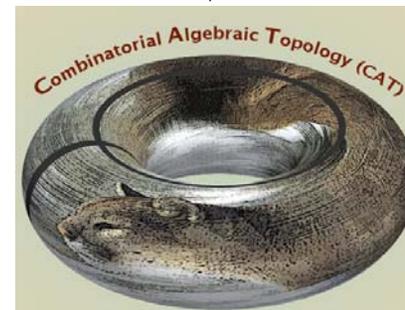
(by invitation only)

29–30 August 2009, Reception Evening 28 August  
Hilton Santa Fe, NM



# CAT Workshop

- **Funders**
  - CSRI, ASC, NNSA, DOE, U.S.A. taxpayer
  - Thuc Hoang (DOE), Njema Frazier (DOE)
  - Scott Collis (SNL)
- **Organizers**
  - Scott Mitchell (idea, proposal)
  - Shawn Martin (logo)
- **Note takers – written summary**
  - David Day
  - Janine Bennett
- **Admin**
  - Deanna Ceballos (financial)
  - Bernadette Watts (website)
- **39 Technical participants**
  - 18 labs: 16 SNL, 1 LLNL, 1 LBNL
  - 20 univ
  - 1 commercial



# Computer Science Research Institute (CSRI) CAT workshop's sponsoring program

Measure	2007	2008
Projects	14	3
Workshops	4	7
Visitors (Institutions)	102 (67)	106 (68)
Summer Students	43	34
Sabbaticals	3	2
% Hires having prior CSRI partnerships	73%	80%



# Computer Science Research Institute (CSRI)

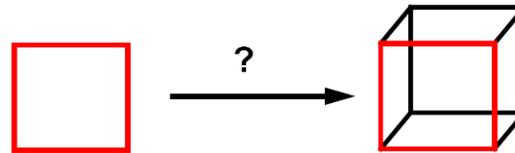
- **Open environment**
  - no guards, gates, guns
  - focus on publications
- **Productive place to visit**
  - faculty
  - post-docs
  - contracts
- **Excellent experience for summer students**
  - graduate and some undergrad
  - extensive overview lecture series
- **Where I work**
  - Scott's NGC story



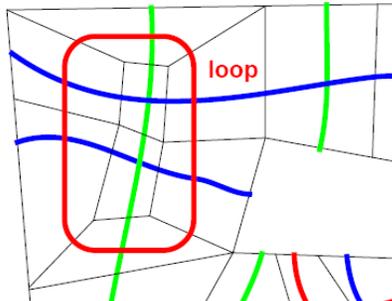


# Since this is a reception talk... “How I got interested in topology”

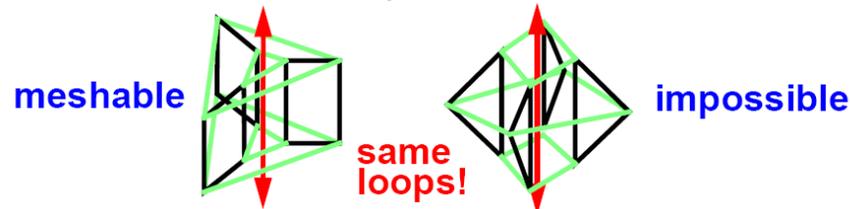
- Algebraic topology Cornell grad-school class – where are the pictures?
  - a doughnut, “applications”, ... tried to forget all that, but it came back...
- Hex mesh generation circa 1990’s
  - Q: Given a quadrilateral mesh enclosing a volume, can you fill volume with hexahedra?



- Homology and hex-mesh existence proof
  - Thurston newsgroup posting 1993. Mitchell STACS paper 1995.
  - **A: Yes.** Dualize surface mesh into arrangement of discrete curves cycles.  
(1993 P. Murdoch rediscovered hex dual = arrangement of surfaces)
    - ball: even #quadrilaterals = necessary and sufficient
    - non-ball w/ embedding in 3d: above +



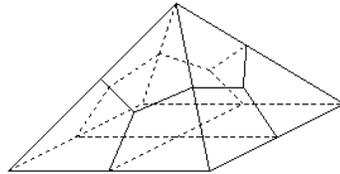
The cycles must have even length if they are null-homotopic in volume



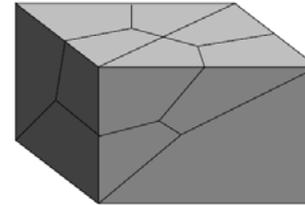
# How I got interested in topology

- Meshing circa 1990's
  - template challenges -> existence proof -> template constructions

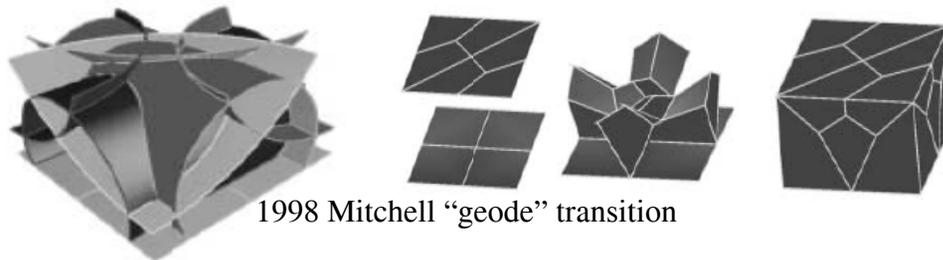
tets diced into hexes  
to semi-structured hex  
transitions



Robert Schneiders's "open problem"

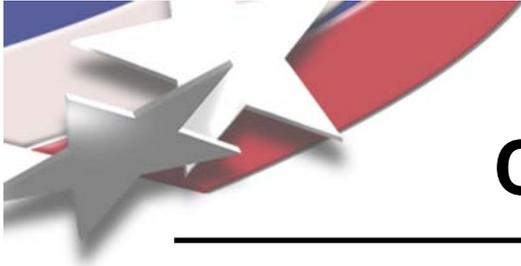


1995 David Eppstein  
hexes =  $O(\text{quads})$   
template transition



1998 Mitchell "geode" transition

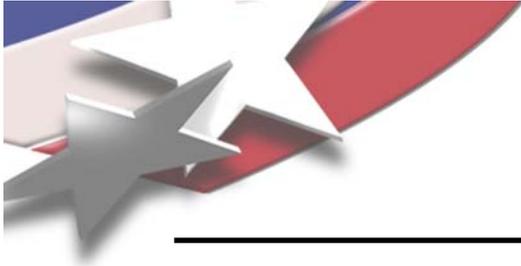
- various challenges for non-ball models -> decomposition by  $H_1$  generators finesses in theory, but no tools to do it
- hex mesh generation via topological arrangements
- hex mesh improvement "swaps"
- I could never get the geometry right for anything to be very useful for finite elements...
- 2000 Project leadership -> 2002 management - plead temporary insanity ☺
- 2007 returned to technical work
- 2008 looked around
  - dozens of SNL apps crying out for discrete combinatorial topological solutions!
  - topology community turned computational!
  - Form a small research team, Shawn Martin, David Day, Eric Boman.
  - Get funding for a workshop! Here we are!



# CAT Workshop Motivation

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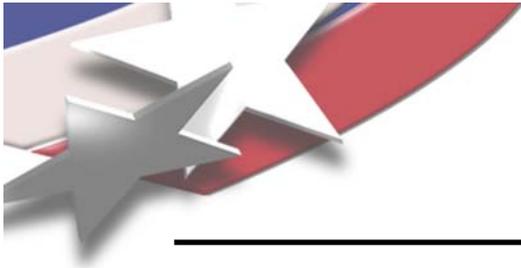
- Time right for a CAT workshop
  - Other events
    - Linear algebra factorization
      - Algebraic Topological Methods in Computer Science "satellite conference" of European Congress of Mathematics, 2008. - **8 speakers**
      - Applied Algebraic Topology Minisymposium at the Fifth European Congress of Mathematics, 2008. - **5 speakers**
      - IMA shortcourse Applied Algebraic Topology 2009, **Carlsson & Ghrist, Henry Adams (also CAT)**
    - Computational Geometry, solid modeling
      - SoCG Symp. Comp. Geom, 2009 had **1-3 topo papers**
      - CCCG Canadian Conf. Comp. Geom, **few papers**
    - Visualization, Morse-Smale theory
      - TopInVis, 2009 Utah **23 talks**, 2007 Germany, 2005 Slovakia
    - Math, Mark Brittenham's "low dimensional topology conference page"
      - **11 annual events** (regional?), **6 sessions at AMS** meetings, **16 "other"** events, 2009  
<http://www.math.unl.edu/~mbrittenham2/ldt/conf.html>
  - Bring together researchers from different communities
  - all new to me... but even established groups within an area meet each other for 1<sup>st</sup> time.
  - Visualization, solid modeling, linear algebra, computational geometry, image analysis,
- Scope is discrete, computation, homology, Morse-Smale
- More CSRI-sponsored workshops possible
  - please send feedback, ideas!



# CAT Workshop Goals

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- **Software, Applications, Algorithms**
  - new univ-labs and univ-univ partnerships
    - many forms possible – opportunities on next 4 slides
      - software partnerships
        - Trilinos for parallel linear algebra *over finite fields*?
      - many SNL topology-ready applications
        - university partnerships to solve?
      - research partnerships
        - univ-labs, univ-univ
  - Identify open problems
    - handle on complexity, scalability
  - Community software plans



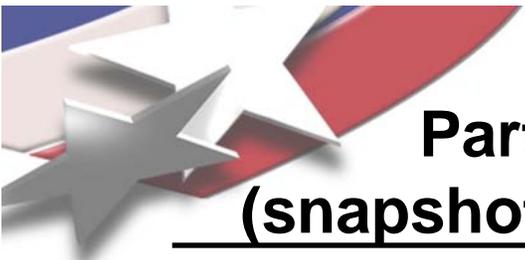
# Software talks

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- **Software**

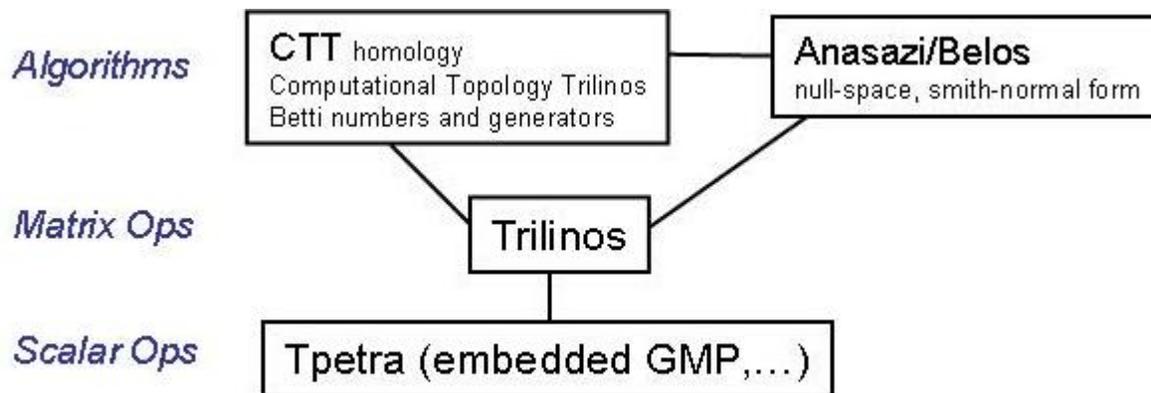
The Saturday morning session focuses on existing software efforts for combinatorial algebraic topology. The intended audience is those wishing to use their tools for research framework or application solutions.

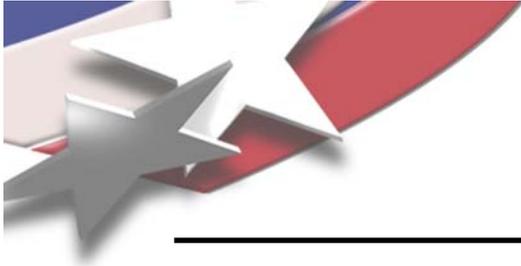
- Speakers are asked to address the following items
  - scope, current and planned
  - capabilities, especially:
    - Betti numbers, homology generators, generators meeting application-specific criteria;
    - filtrations & Reeb graphs for sensitivity and transients;
    - Smith Normal Form factorization, other linear algebra capabilities
  - scalability
  - software maturity/usability
  - availability and usage models
  - please try to limit time on reduction, sampling, and initial complex generation
- 35 minutes = 25 talk + 10 discussion



## Partnership opportunities, software (snapshots, not a workshop program overview)

- **SNL topology-ready software**
  - **Trilinos large-scale parallel linear algebra**
    - general purpose, templated types
    - Heidi Thornquist
  - **Workshop answers whether either of these make sense:**
    - Trilinos in LinBox or JPlex?
    - Trilinos over finite fields?





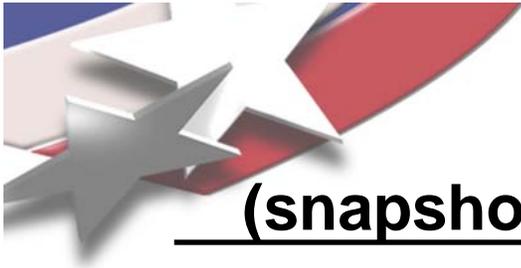
# Application talks

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- **Applications**

The Saturday afternoon session focuses on laboratory applications, especially those that are not currently using topology, but could. Format consists of an overview talk, together with a discussion session to explore what is possible. The intent is for application owners to engage topology experts for help in solving their problems, with the potential for longer term partnerships.

- Speakers are asked to address the following aspects of their applications:
  - geometry, if any
  - dimension: 3d, higher-d, arbitrary-d
  - questions needing solution methods
  - interesting structural features one would like to discover and compute
- 40 minutes = 25 talk + 15 discussion



# Partnership opportunities (snapshots, not a workshop program overview)

- SNL topology-ready applications  
**Red = talk or person at CAT Workshop**

- Morse-Smale for understanding combustion science

- J. Chen, Ray Grout, Valerio Pascucci, Janine Bennett, David Thompson, ...

- Fracture and fragmentation in meshes

- Alejandro Mota

- Foam material analysis

- L. Romero

- Solid Model decomposition and parameterization, defeaturing

- Tamal Dey (non-SNL). also SNL apps

- Sensor networks;

- R. Ghrist (U. Penn). also SNL apps

- Topology in 2d image analysis

- Kurt Larson, Carl Diegert

- Discrete combinatorial optimization fitness landscapes (tentative)

- Jean-Paul Watson, Shawn Martin

- Manifold cutting on system of loops for dimensional reduction, in e.g. image analysis x-ray tomography, molecular conformations

- Shawn Martin

- Text analysis parameter sensitivity

- Daniel Dunlavy

- Critical infrastructure network security, Green Grid design

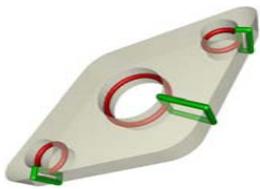
- W. Hart

- Transportation planning

- C. Phillips

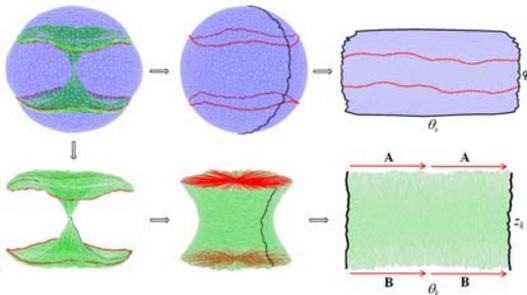
- Radar and signals analysis

- Michael Robinson (non-SNL). also SNL apps



Tamal Dey gen  $H_1$

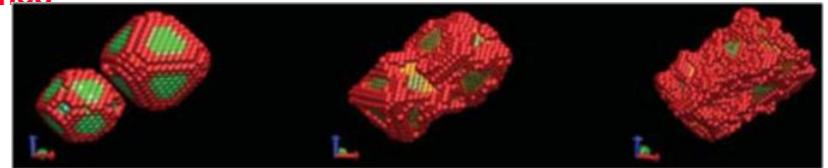
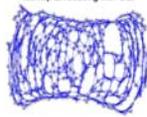
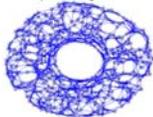
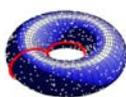
Shawn Martin molecular conformation dim. red.



Torus Data with Cut

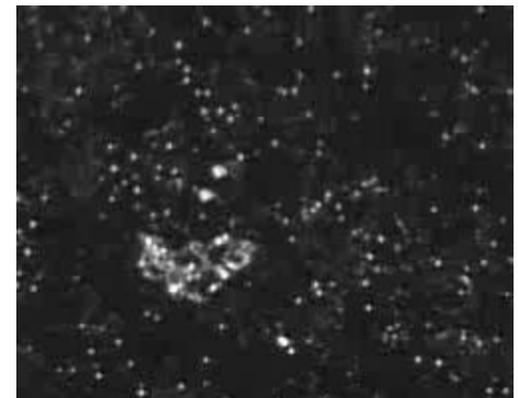
Isomap Embedding without Cut

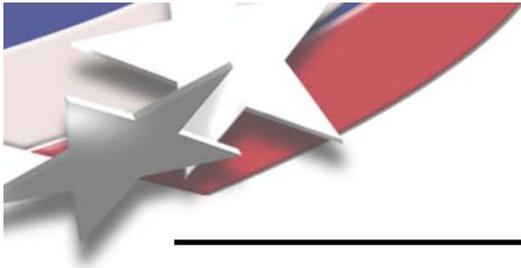
Isomap Embedding with Cut



void coalescence graphic from Marian et al. A. Mota

Reflective Particle Tag. K. Larson, C. Diegert





# Algorithm talks

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- **Algorithmics**

The Sunday morning session focuses on algorithmic challenges. The intended audience is those already familiar with the algorithm basics, rather than application owners.

- Speakers are asked to address one or more of the following aspects of their algorithmic approaches:

- *algorithmic complexity*, including dependence on genus, dimension, number of vertices, number of simplices, coefficient ring, filtration size, and number of critical points.

- What's the hope for explicit bounds, tight bounds, and improvements?

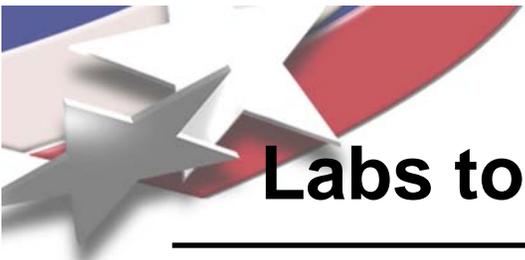
- capabilities for sensitivity analysis and transient features, including Reeb graphs, filtrations, and new math structures

- application-tailored solutions, e.g. homology generators with specific geometry or cardinality; cycle homotopies

- visualization techniques that use topology

- visualization techniques for understanding topology

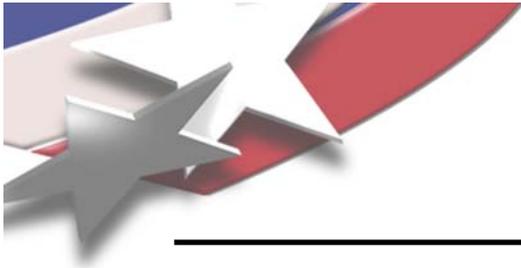
- 35 minutes = 25 talk + 10 discussion



# Labs topology research & development

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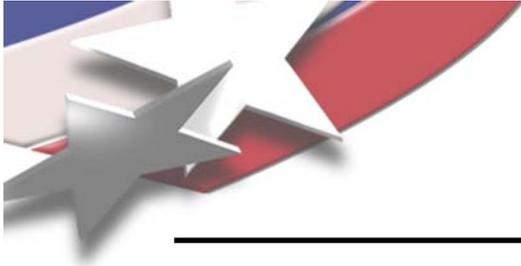
- **SNL**
  - **Topology for Statistical Modeling of Petascale Data, Jacobi Sets,**
    - DoE Office of Science funded 2009
    - P. Pebay, Janine Bennett, David Thompson, M. Rojas, Valerio Pascucci
  - **Computational Topology, ASC, homology algorithms and applications**
    - Scott Mitchell, Shawn Martin, David Day, Erik Boman, Janine Bennett
    - Optimization manifold exploration, dimensional reduction, molecular conformations
      - Jean-Paul Watson, Shawn Martin
  - **Visualization tools, SNL-Univ. Utah partnership to put V. Pascucci's capabilities in SNL VTK/Titan framework**
    - **Jason Shepherd, Valerio Pascucci**
  - ongoing proposals ...
- **LLNL**
- **LBNL**



## Panel - summarize

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- David Saunders  
Peer-Timo Bremer  
Dmitriy Morozov  
Shawn Martin  
Michael Robinson
- Questions
  - Rank applications in terms of low hanging fruit that topology can pluck.
  - What are the key open problems, or main roadblocks, for advancing algorithms?
    - In particular, comment on scalability, and techniques for high dimensional data. In particular, comment on generalizations of filtrations
  - What new software or software mechanisms/structure would most benefit the community?
    - "New software" means, what techniques would be valuable to have in accessible and general purpose format such as LinBox and Plex? "mechanisms/structure" means, would an open-source effort be helpful? And should general and available versions be developed of Reeb graphs, complex generation methods, or anything else in particular?



# Conclusions

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- **You are at 7,000' elevation**
  - May be hard to sleep
  - Wear sunscreen
  - Drink lots of water
  - Enjoy the guacamole!
- **Talk, discuss, meet, participate, brainstorm!**
- **Slideshow**

