Multiparameter Continuation

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Implicitly defined "surface":

$$F(u)=0 \quad F:\mathbb{R}^{n} \to \mathbb{R}^{n-k}$$

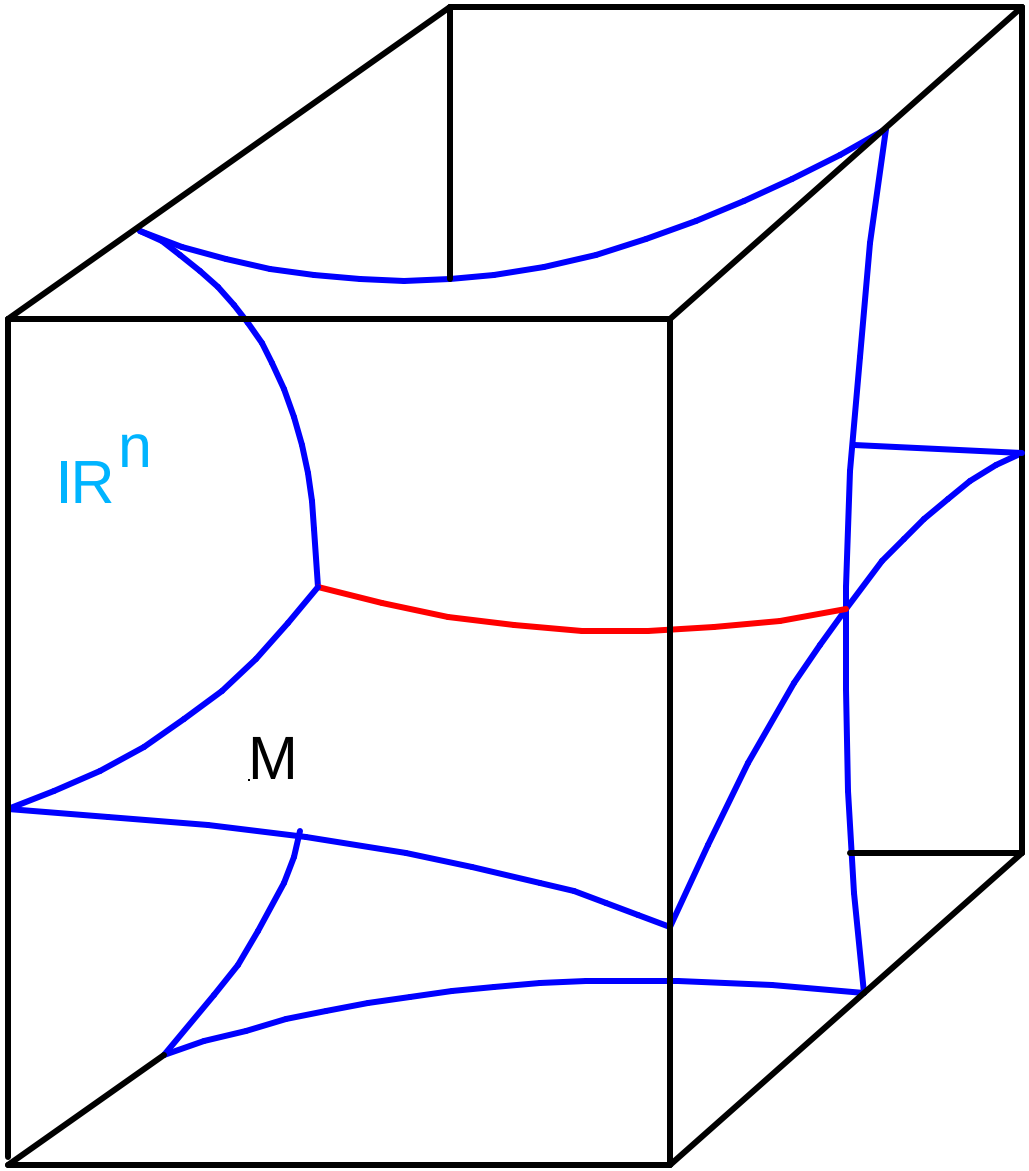
u might be (x,p), with

x a steady state
or periodic motion

p a list of free parameters

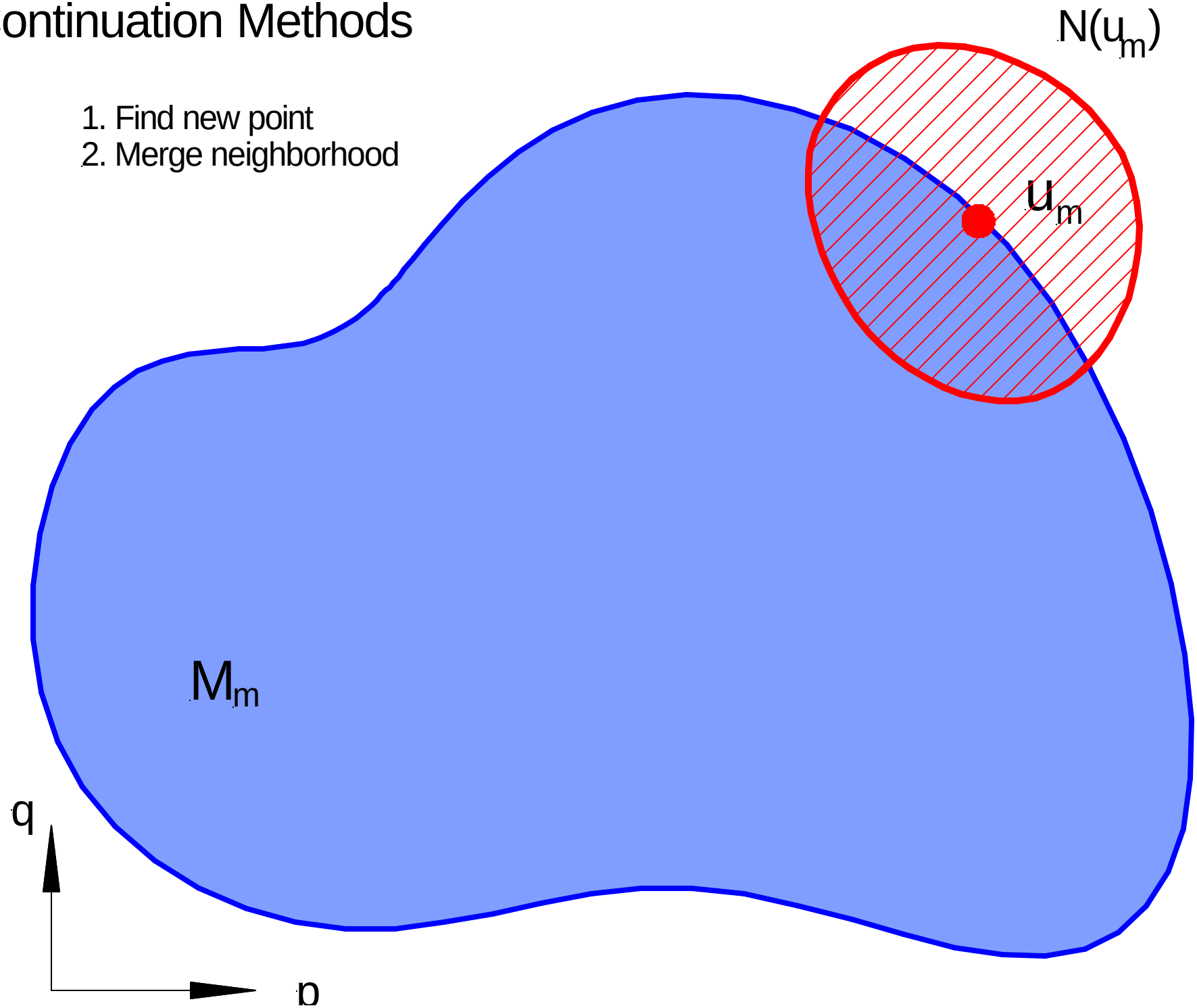
Solution manifold -- pieces of k-dimensional "surfaces" glued together along singular k-1 dimensional "curves".

$F(u)=0 \quad F:IR^{n} \rightarrow IR^{n-k}$

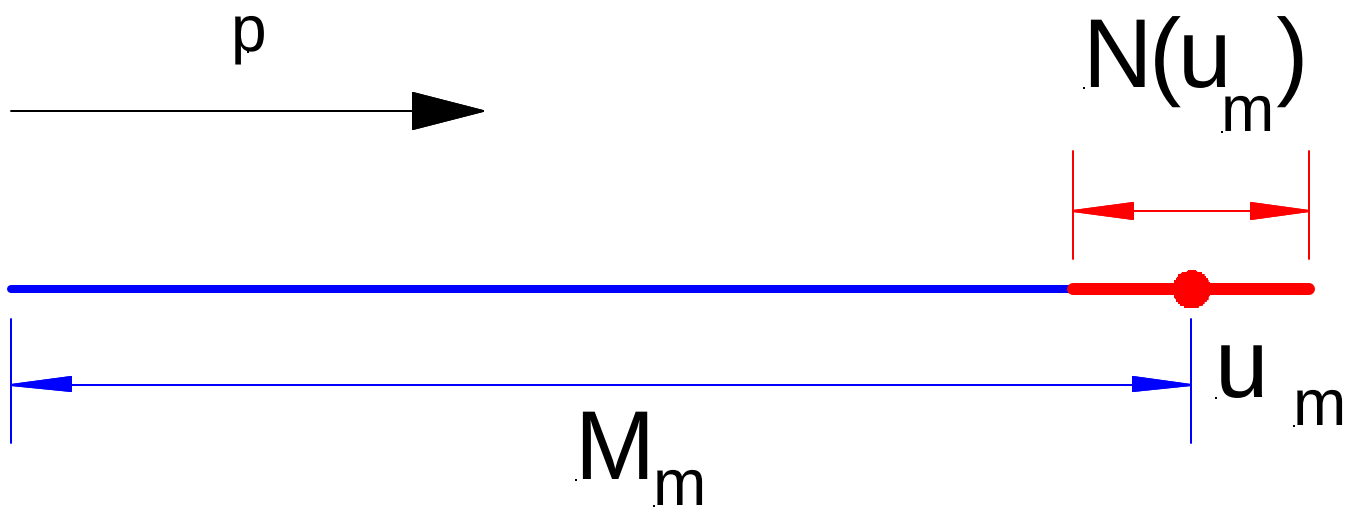


Continuation Methods

1. Find new point
2. Merge neighborhood



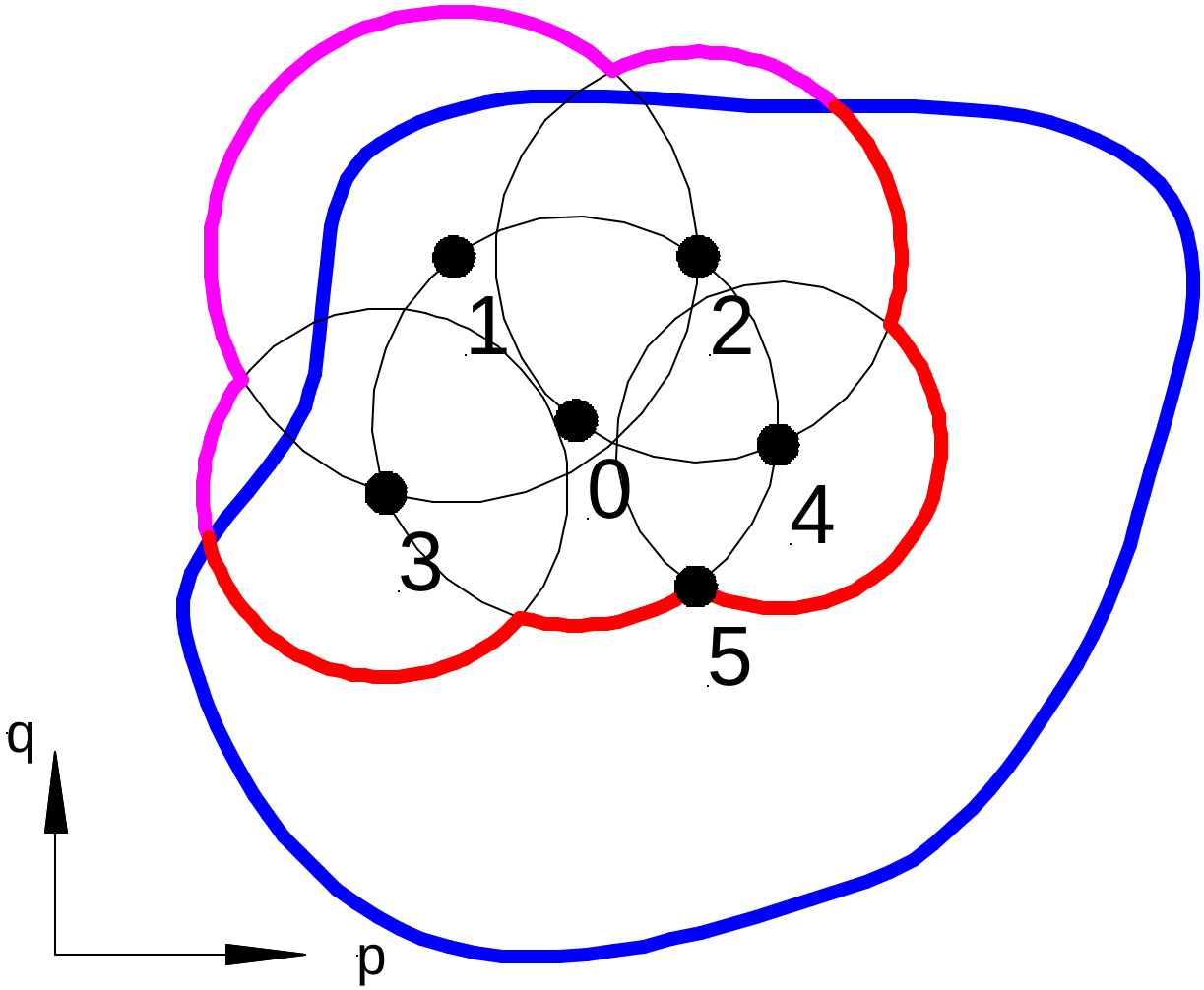
One dimension (*k*=1) is simple



e.g.

How to generalize to *k*>1?

$k>1$



Methods differ in how they represent the M_m.

This choice makes finding a point and merging harder or easier.

Using a mesh make new point easy but merge hard.

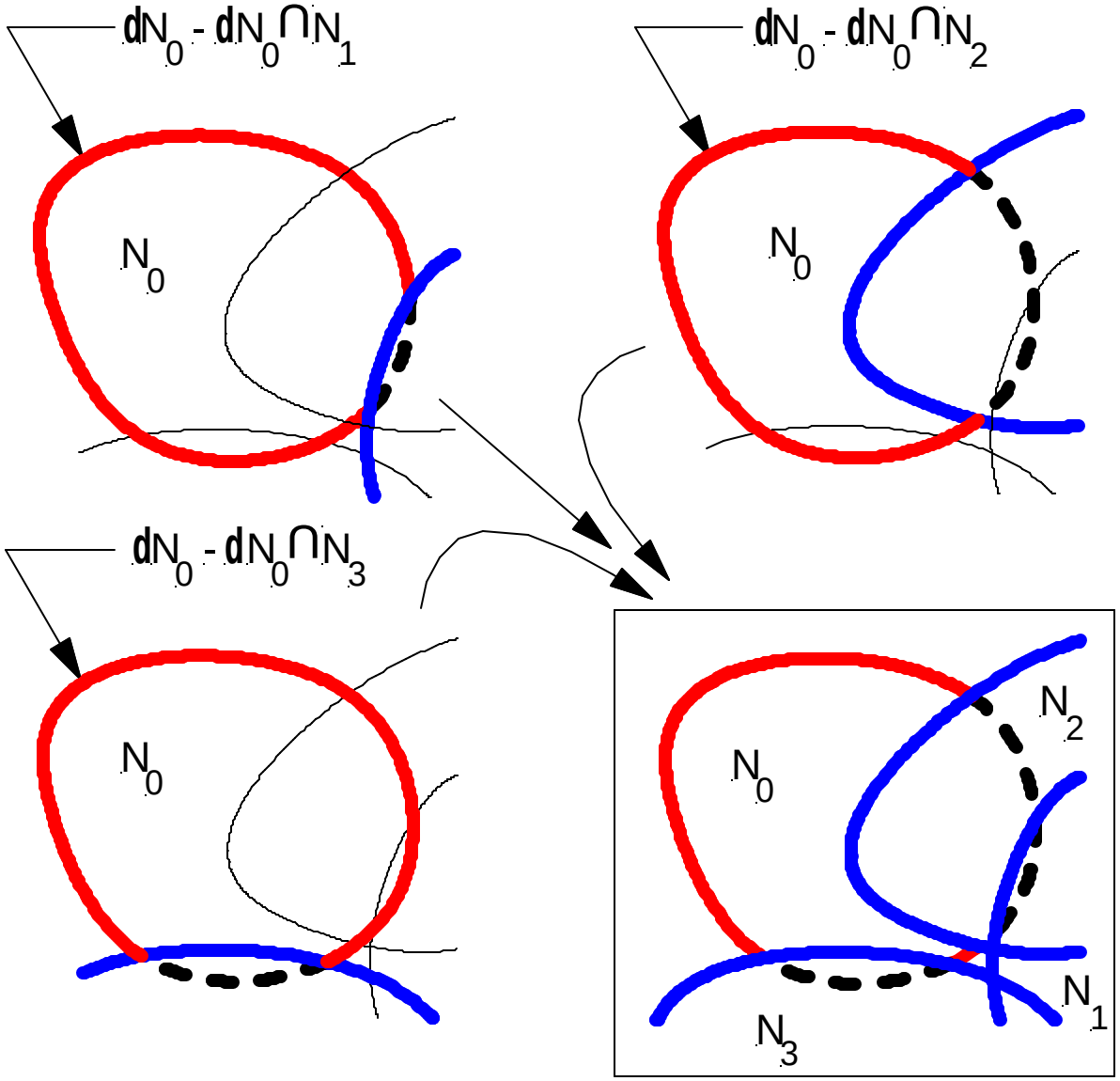
Using a covering makes merge easy but new point "hard".

Select new point from boundary in **W**.

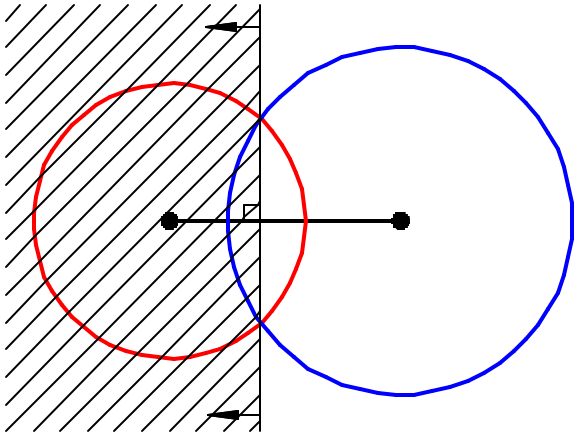
Volume covered keeps increasing.

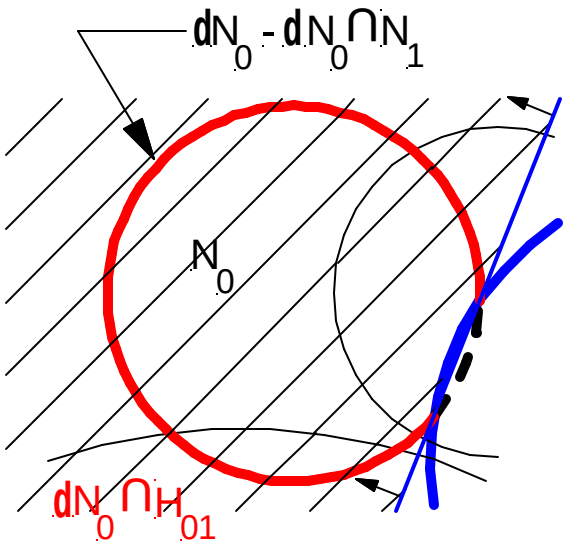
The boundary of a union of neighborhoods - *points not interior to any N_i*

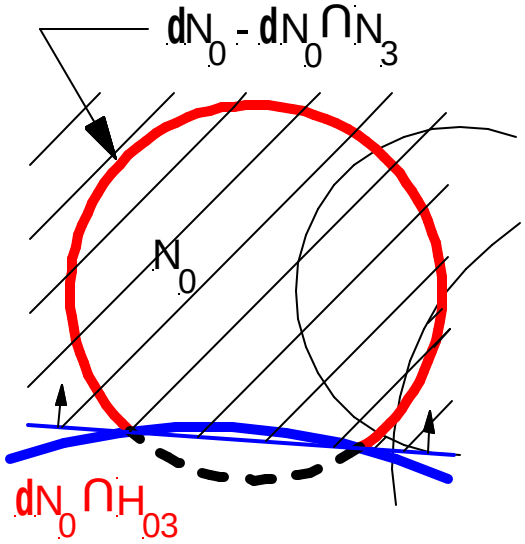
Can form the boundary from pairwise subtractions

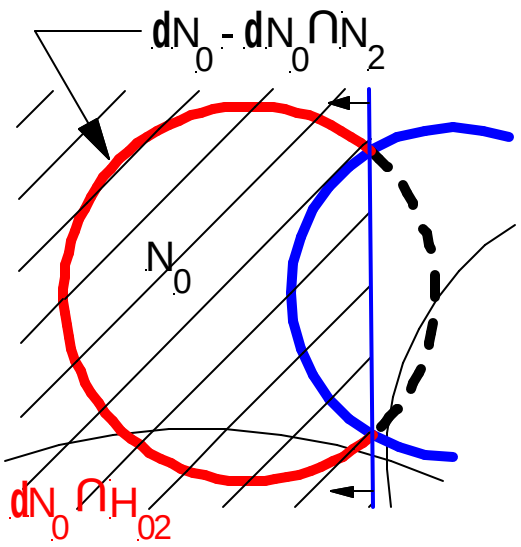


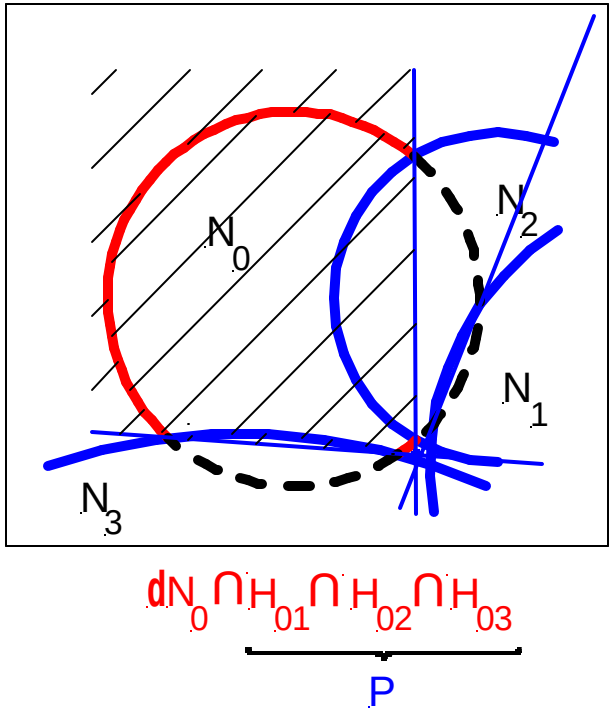
If N_i is a spherical ball





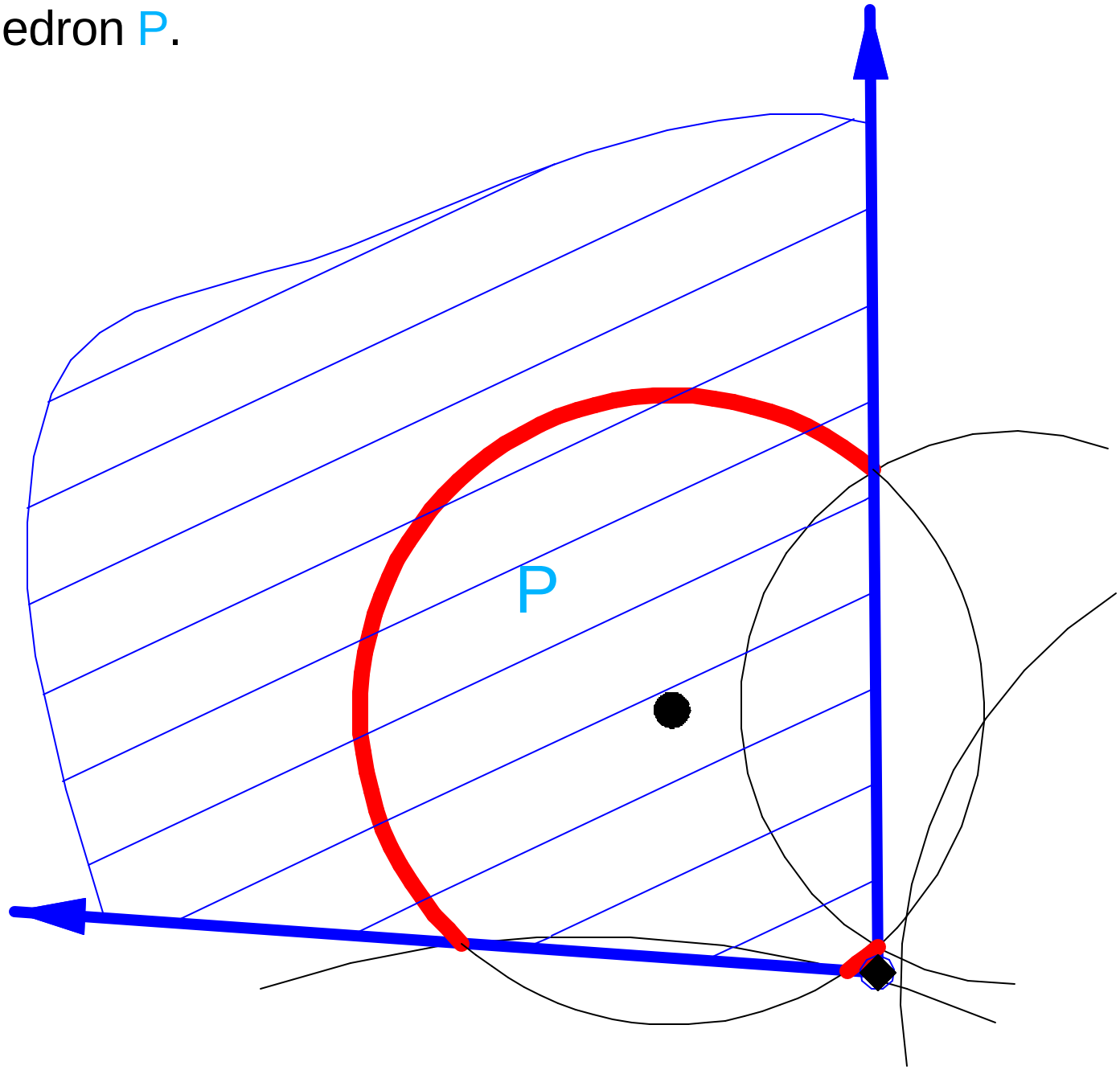




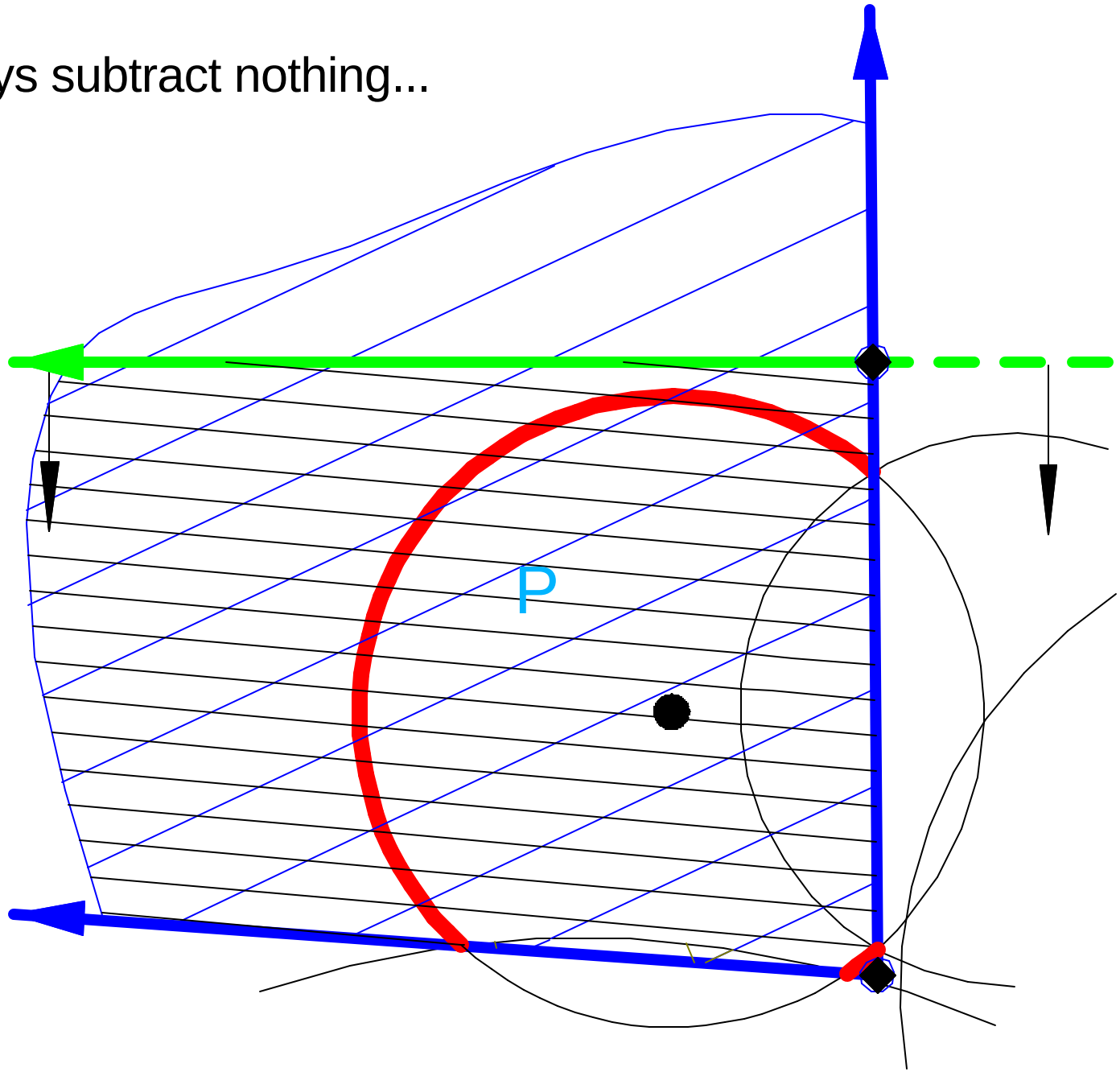


Instead of doing the subtractions can use the blue polyhedron P.

P is convex

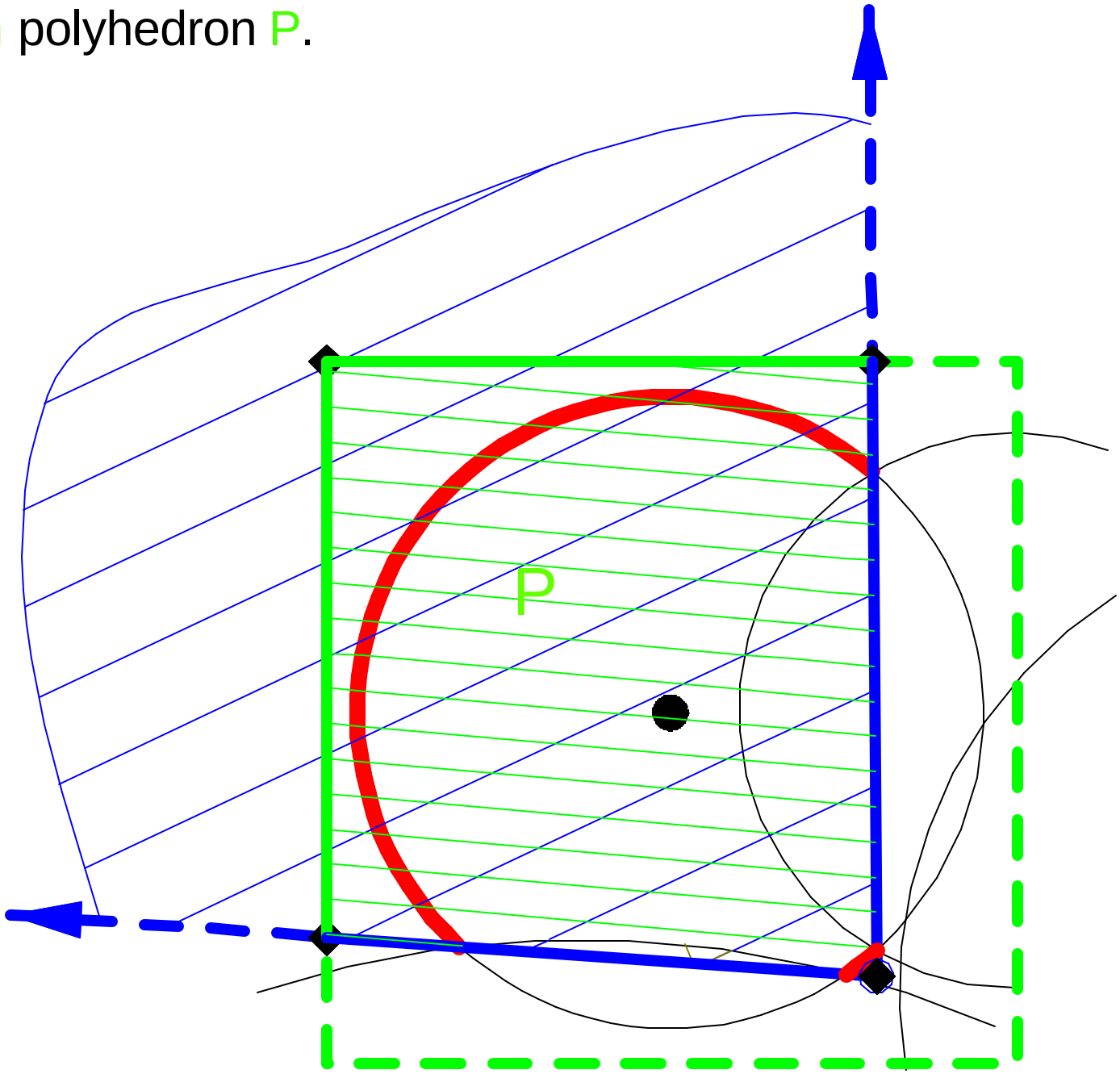


Can always subtract nothing...



Instead of doing the subtractions can use the ~~blue~~ green polyhedron P.

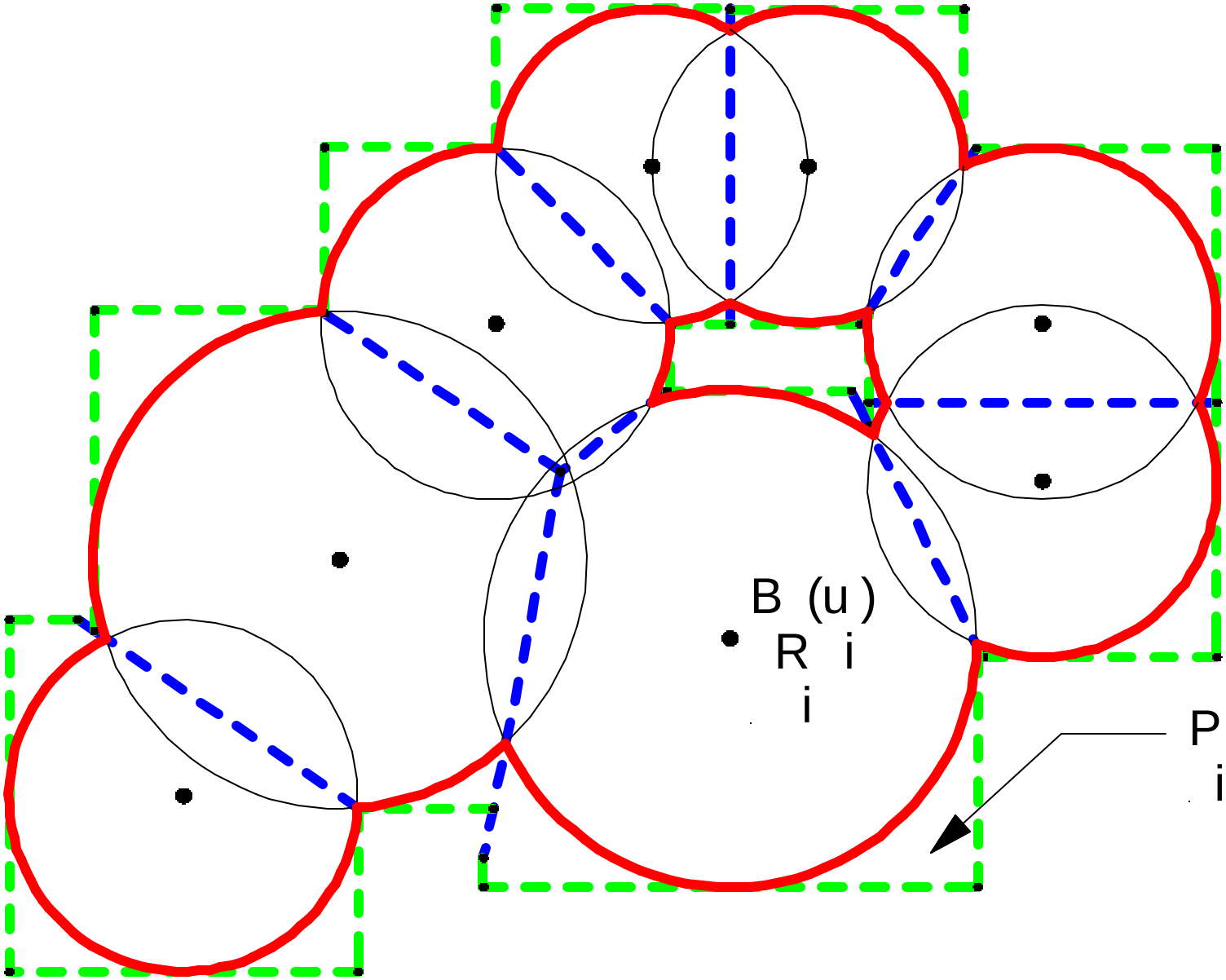
P is finite, convex



Boundary -> on Sphere and in Polyhedron

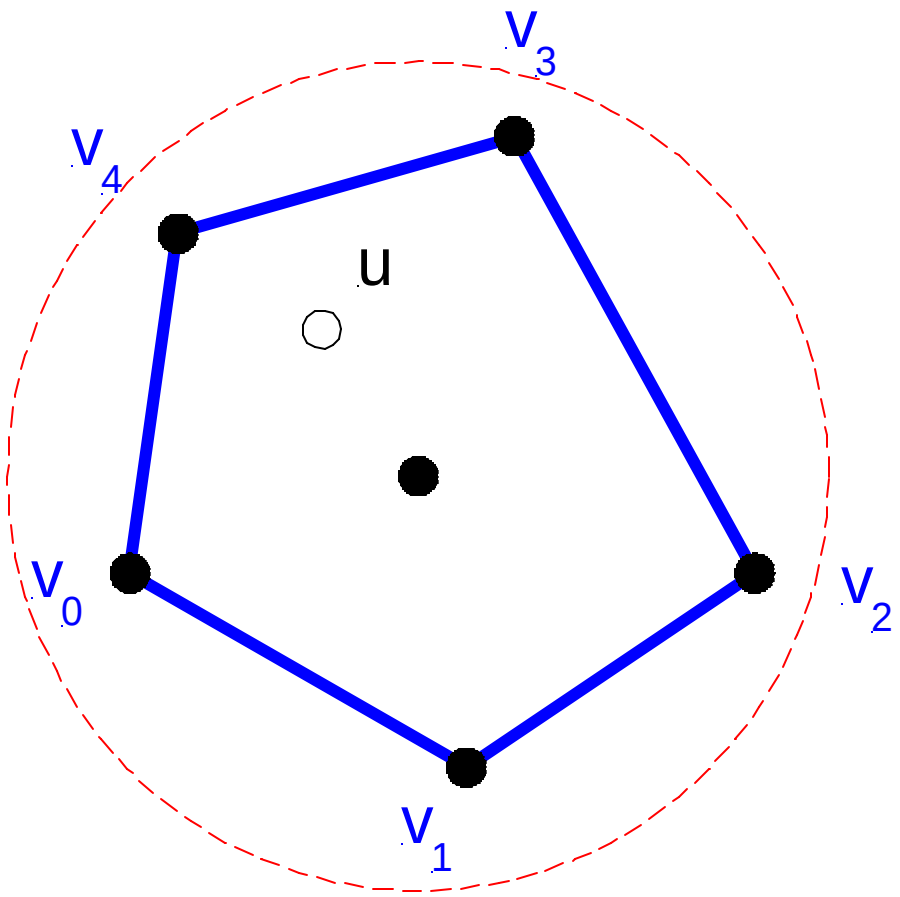
Power Diagram a.k.a.
"Restricted Laguerre Voronoi Diagram"

Edelsbrunner



Finding a point on the boundary

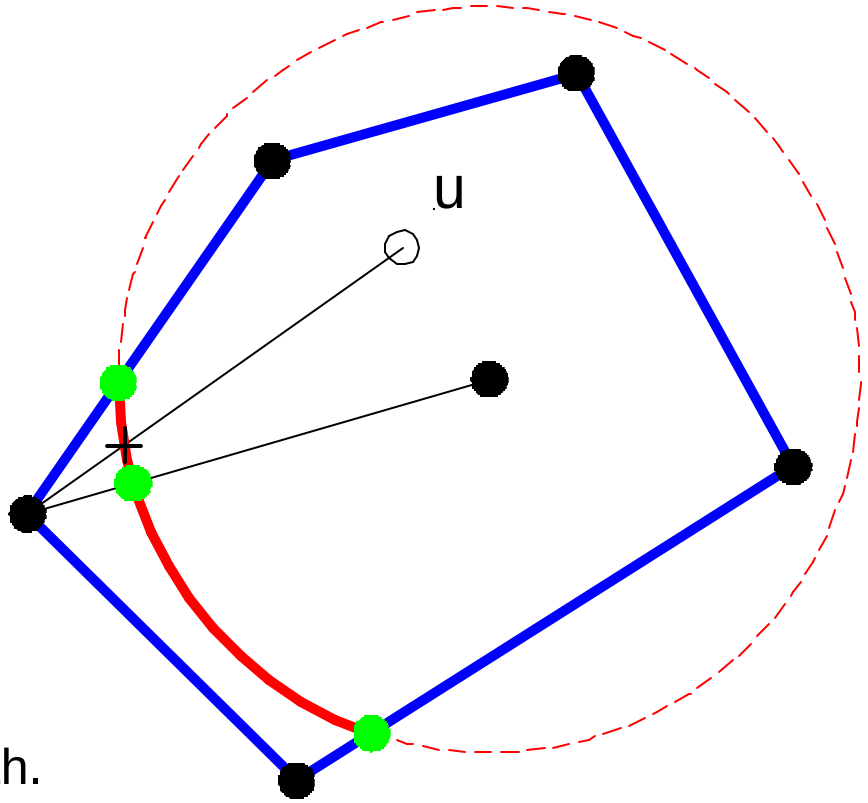
If all vertices of the polyhedron lie inside the ball --



no contribution to boundary of union.

Finding a point on the boundary

If a vertex of the polyhedron lies outside the ball --



"All" we have to do is find a point u in both.

Edge that crosses.

-- or --

If ratio of radii <sqrt(2) center is in P and can use the origin.

Continuation

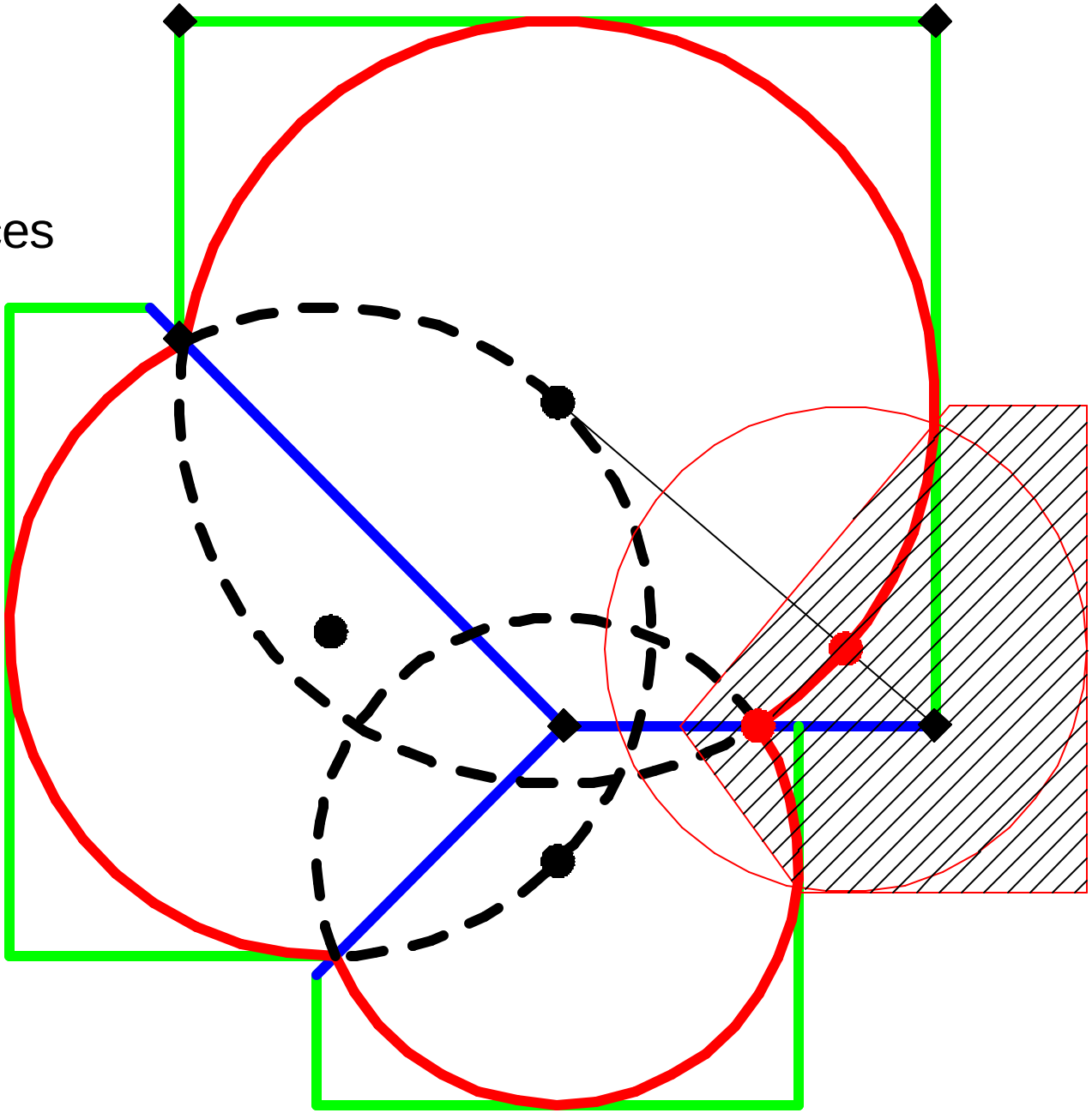
Find a P w/ ext. vert.

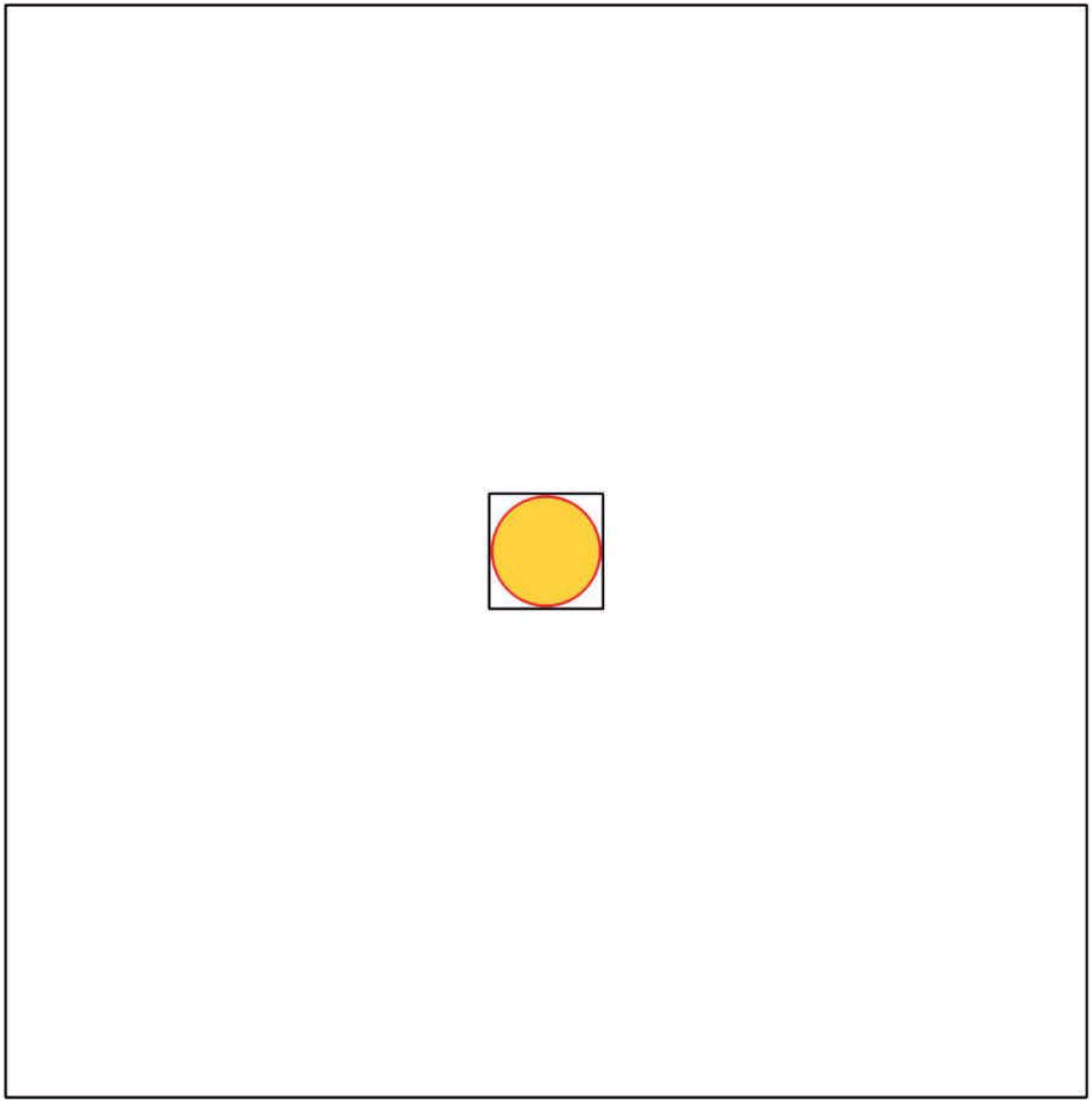
Get pt. on dM

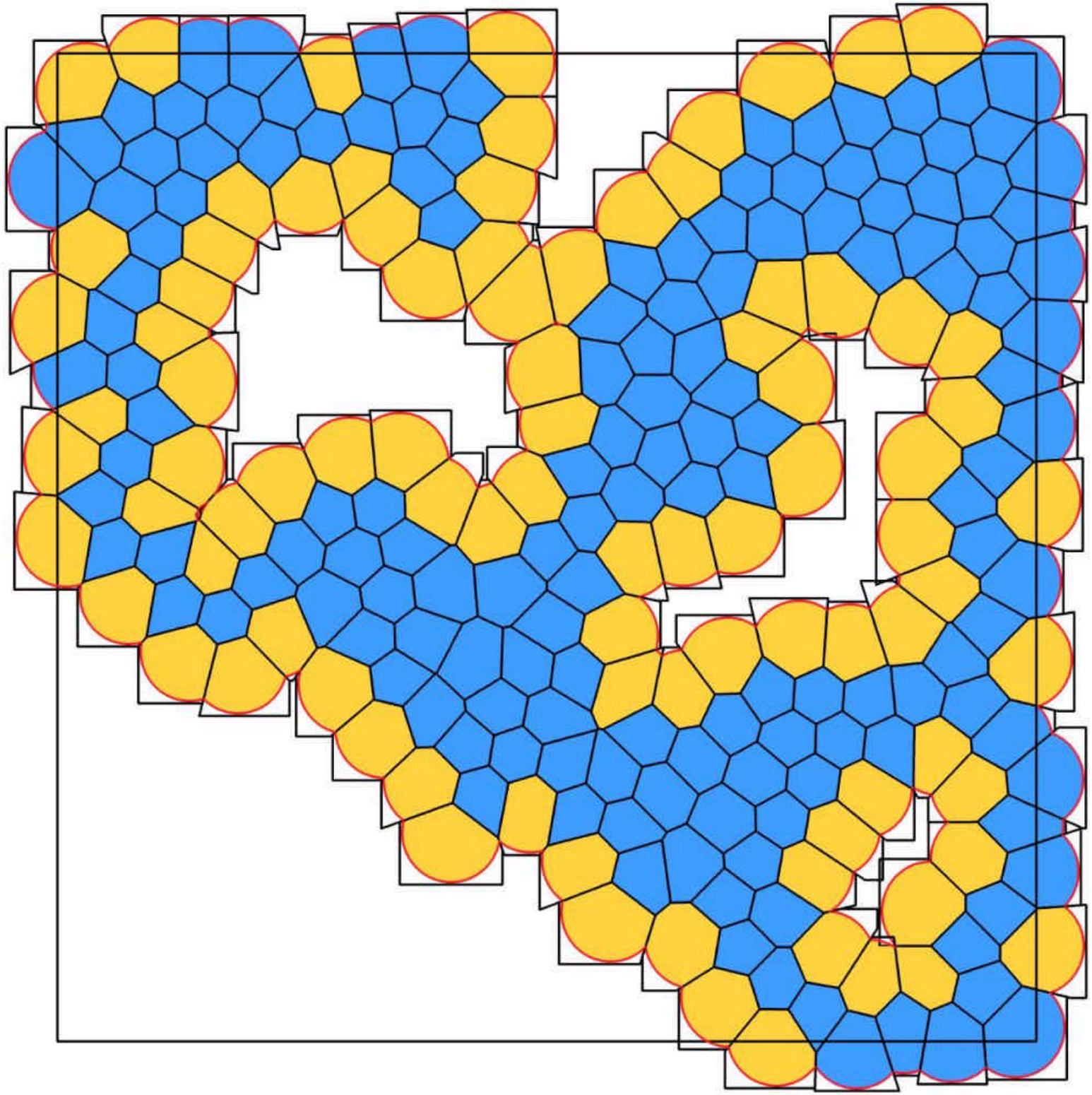
P=cube

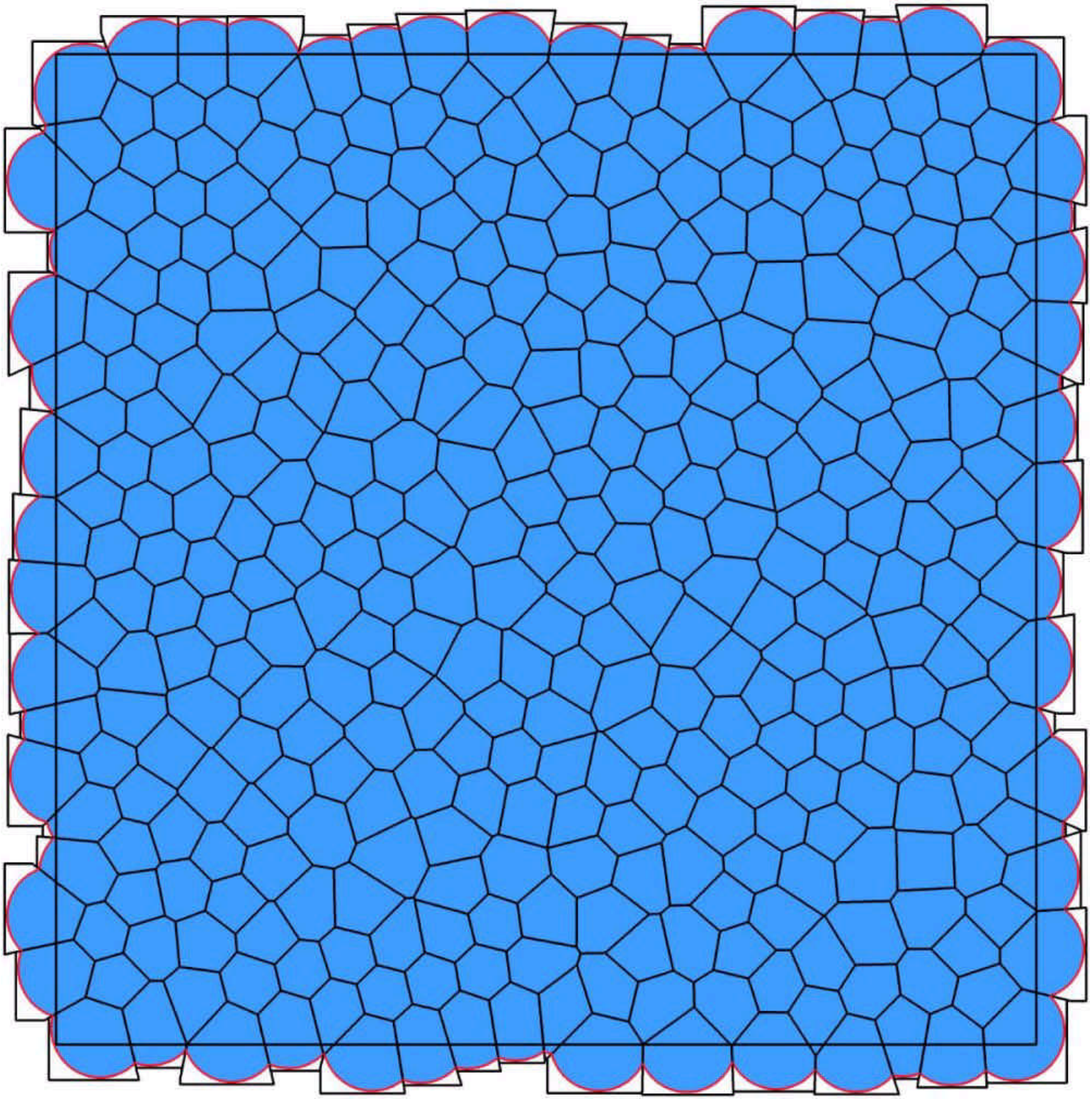
Find overlaps

Remove 1/2 spaces

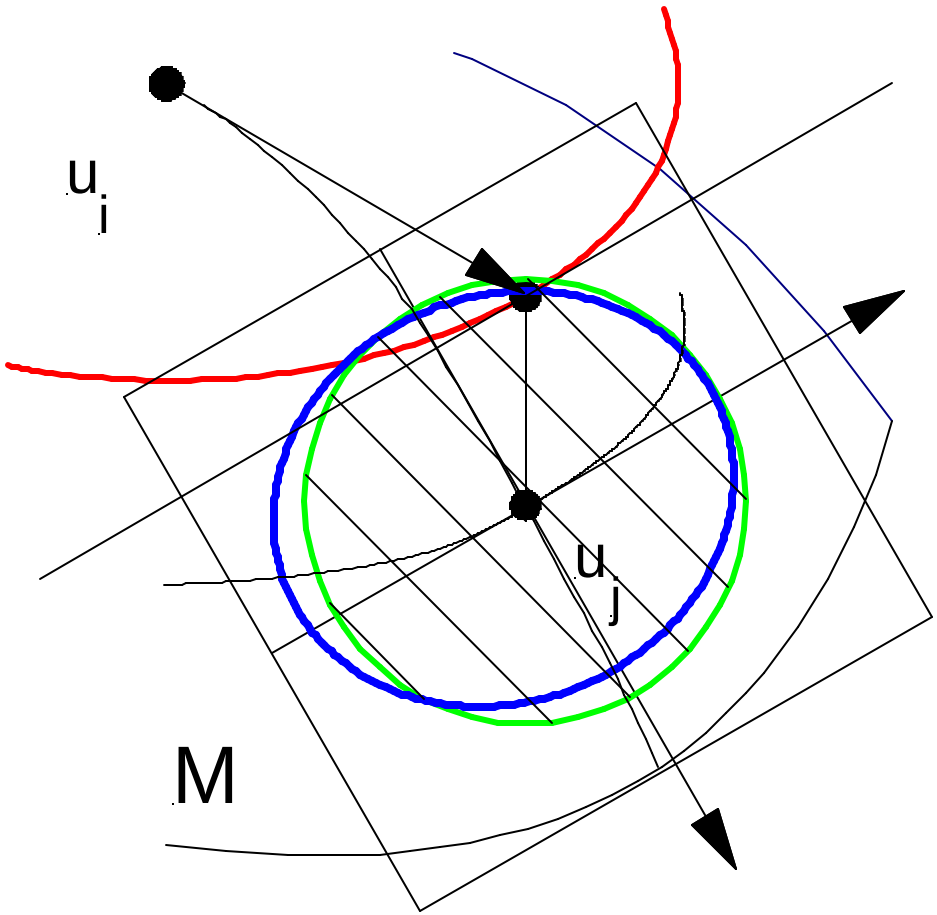


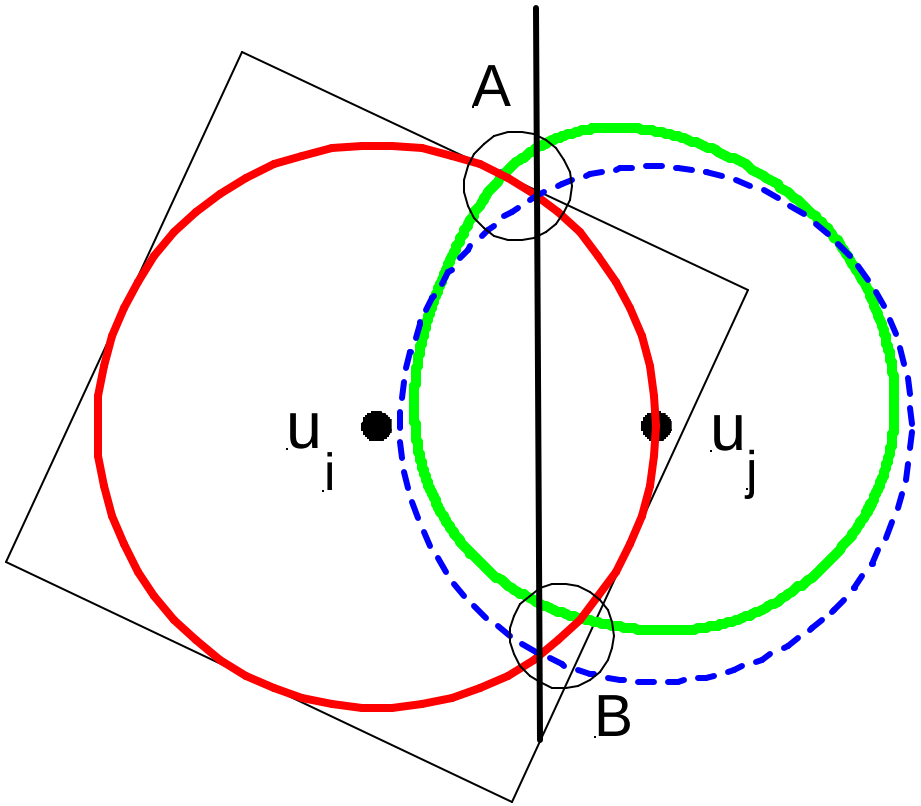




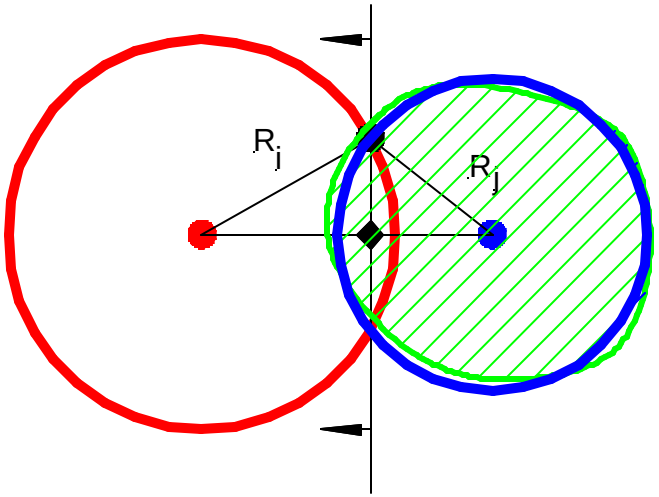


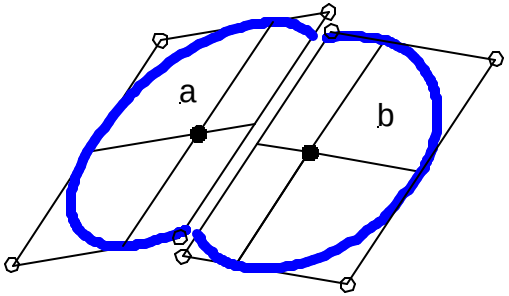
Surfaces that aren't flat -- Charts



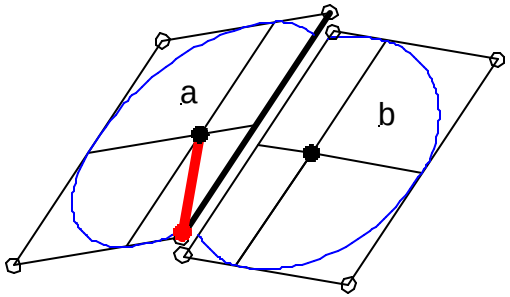


Adding a new ball when M not flat.



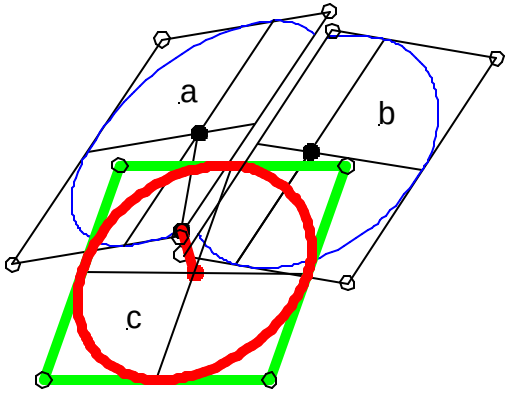


1. Initial

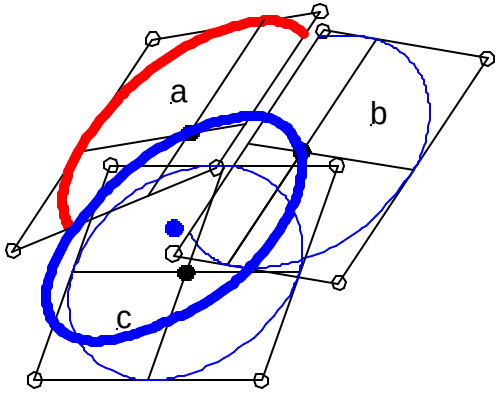


2. Find Pnt on Bnd

Norm in IR^k

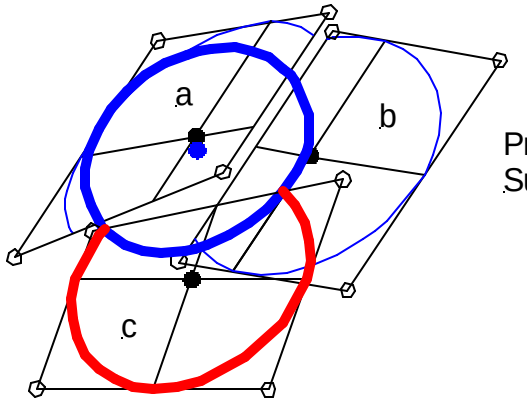


3. Project, create new ball/TS



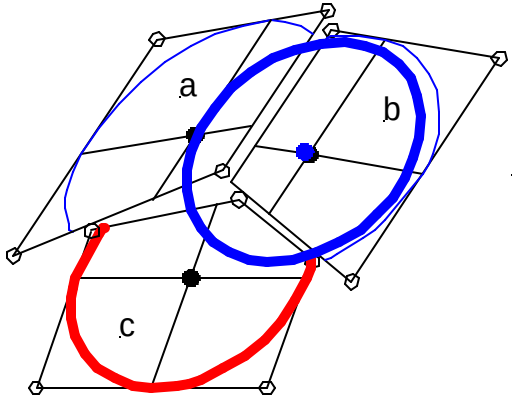
4. Subtract c from a

Project center onto k-space,
Subtract half space



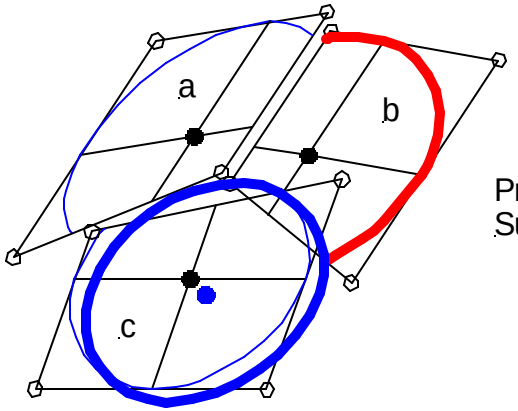
Project center onto k-space,
Subtract half space

5. Subtract a from c



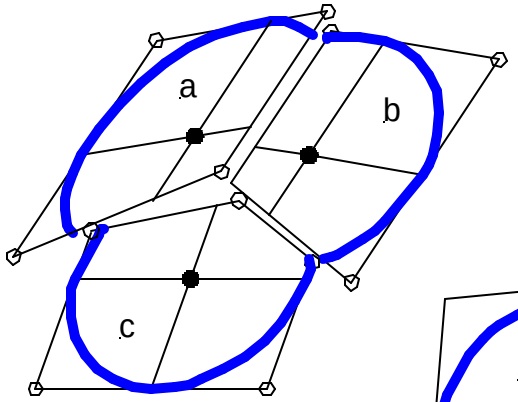
Project center onto k-space,
Subtract half space

7. Subtract b from c

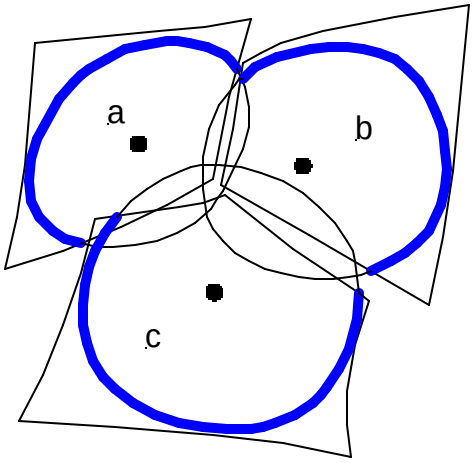


Project center onto k-space,
Subtract half space

6. Subtract c from b



8. Final

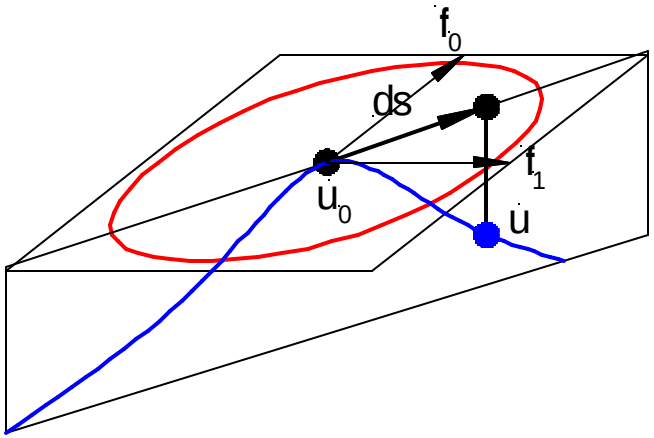


All operations basic linear algebra
plus subtract 1/2 space from polyhedron

Need:

Project from TS to M
Find TS.

Project a point onto M



$$F(u) = 0$$

$$\mathbf{F}^T(u - u_0) = ds$$

k pseudo-arclength constraints.

Find Tangent space

$$\mathbf{F} = \text{Null}(F_u(u))$$

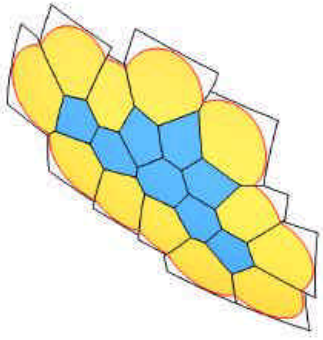
F is the matrix whose columns are **f**_0, **f**_1, ..., an orthonormal basis for the tangent space

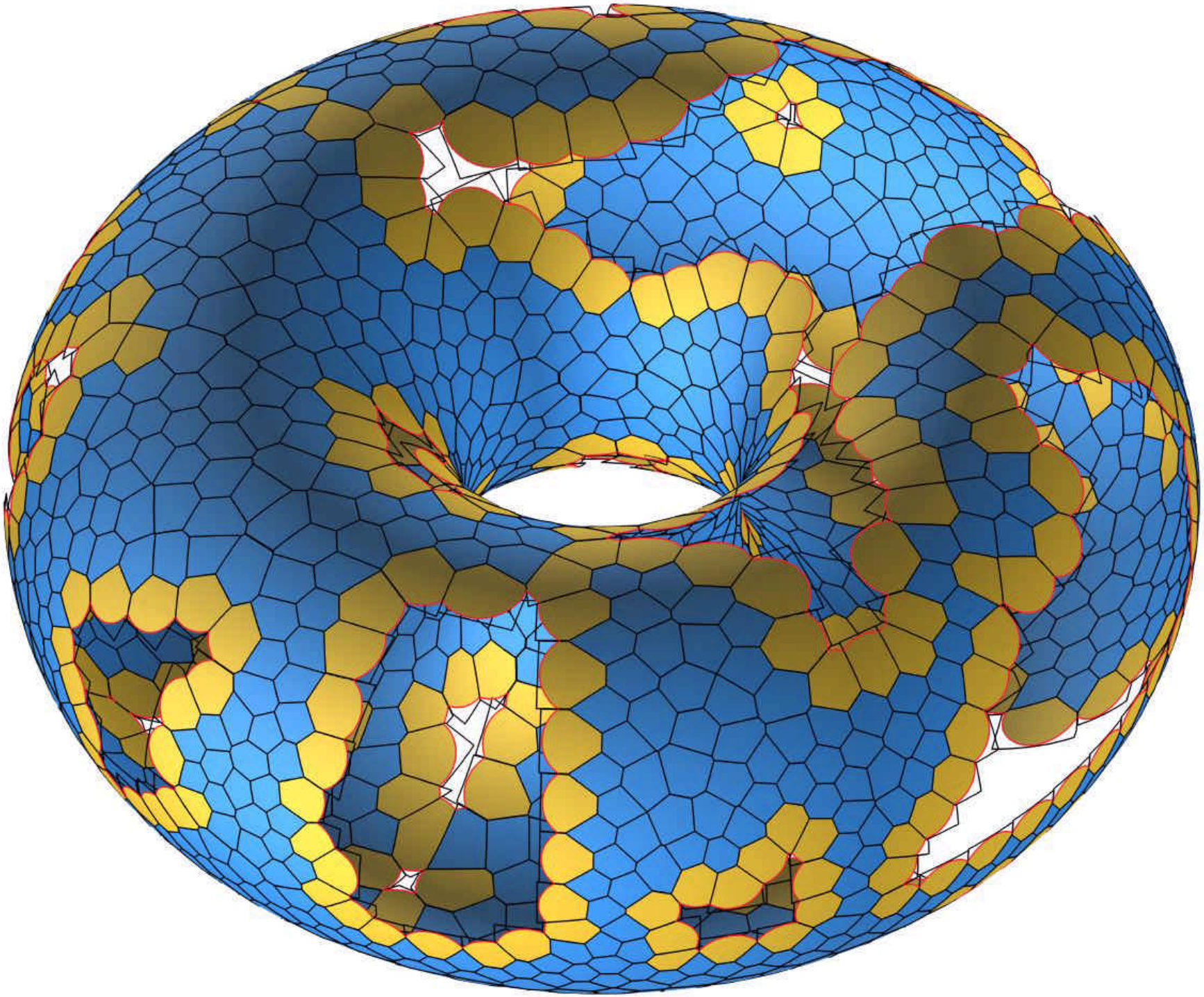
ds is a *k* vector

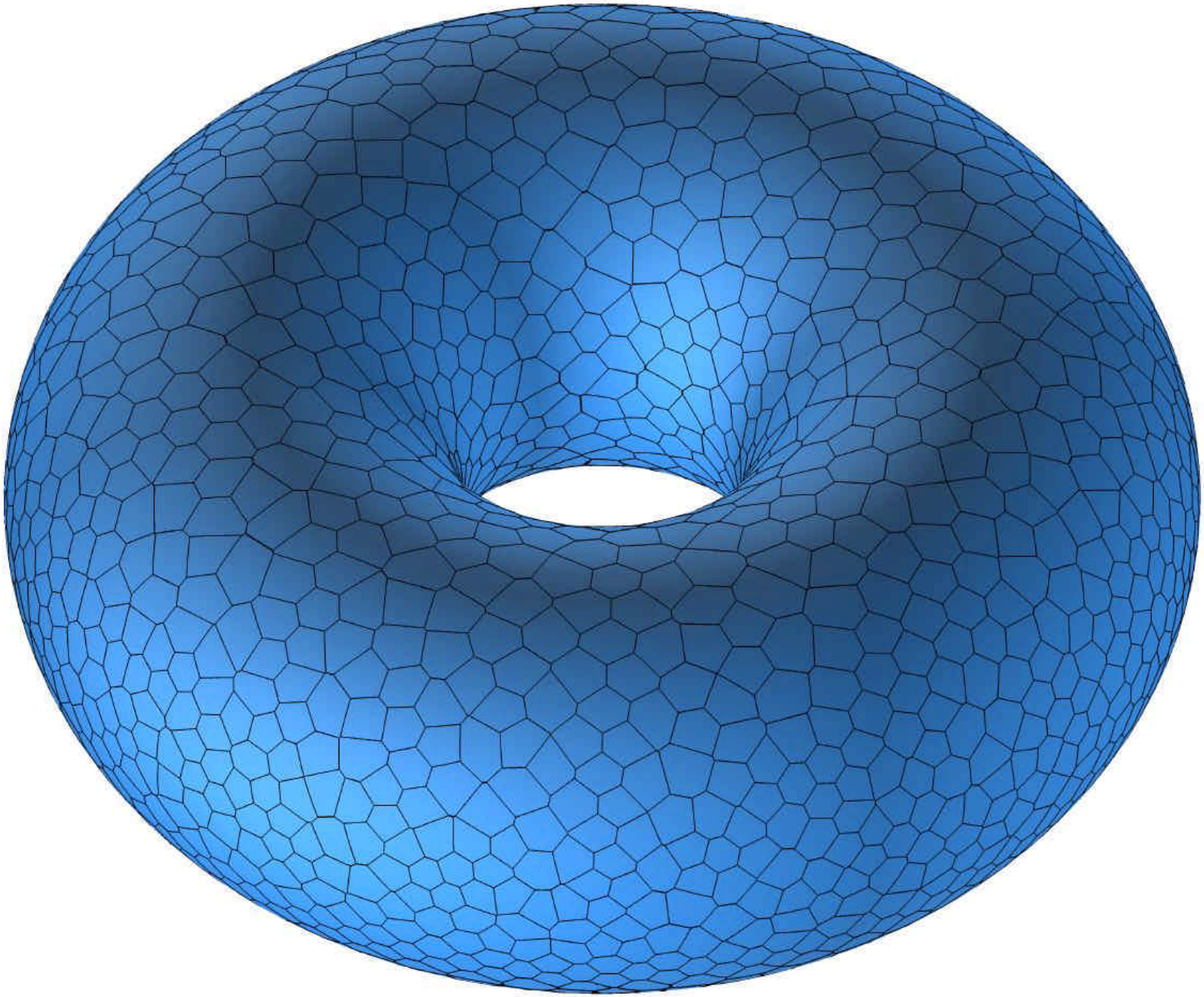
$$F_u(u)\,\mathbf{F} = 0$$

$$\mathbf{F}_0^T\mathbf{F} = I$$

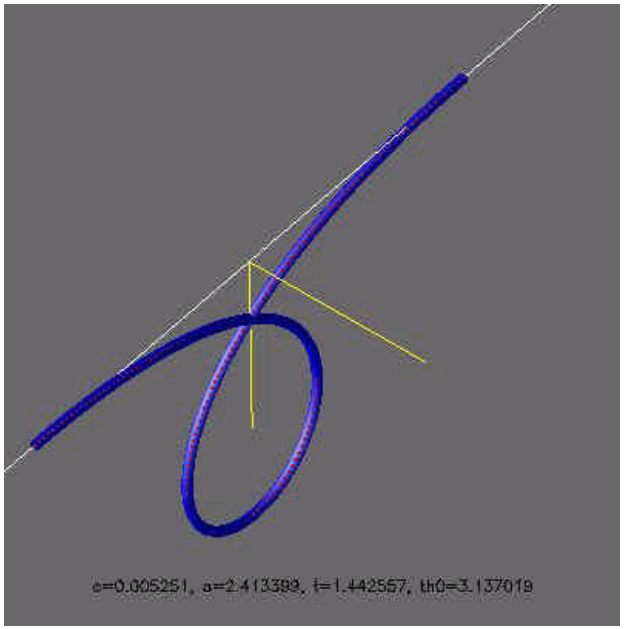
Then orthonormalize.

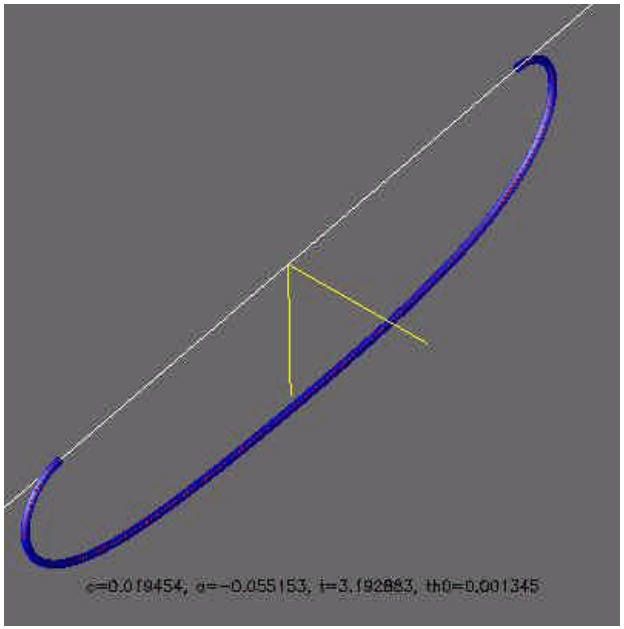


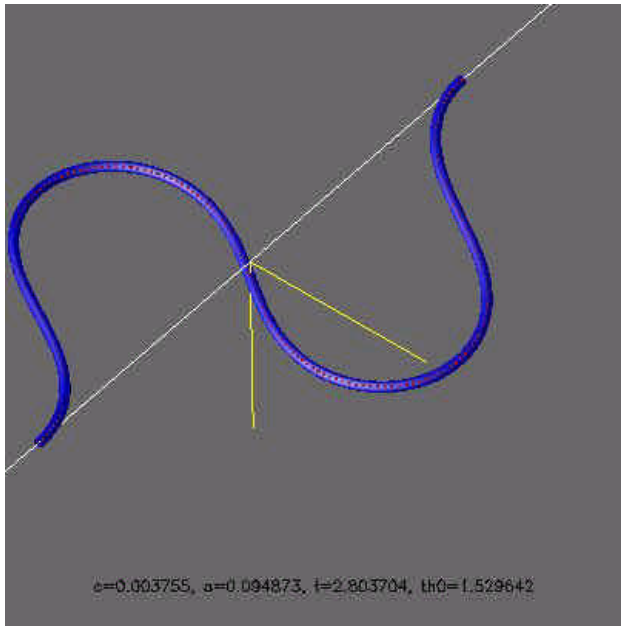


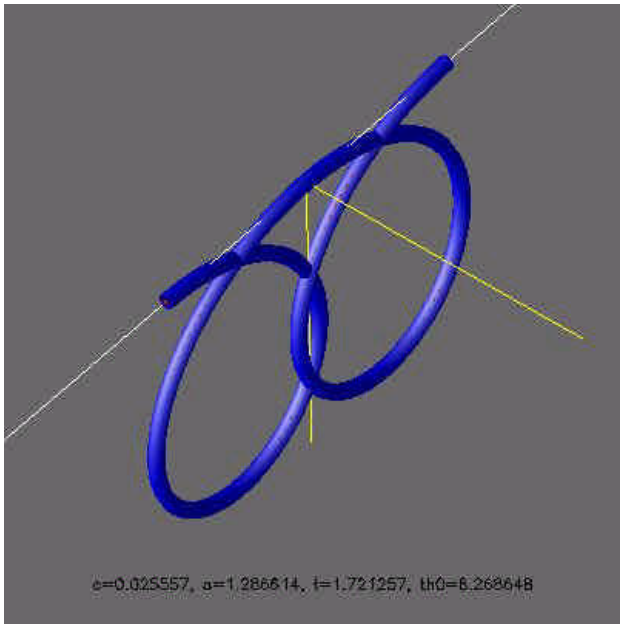


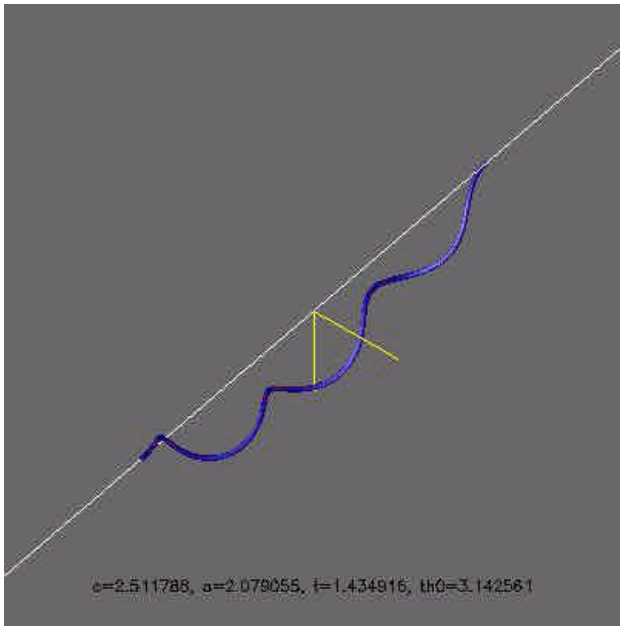
Twisted Rod
Sebastien Neukirch

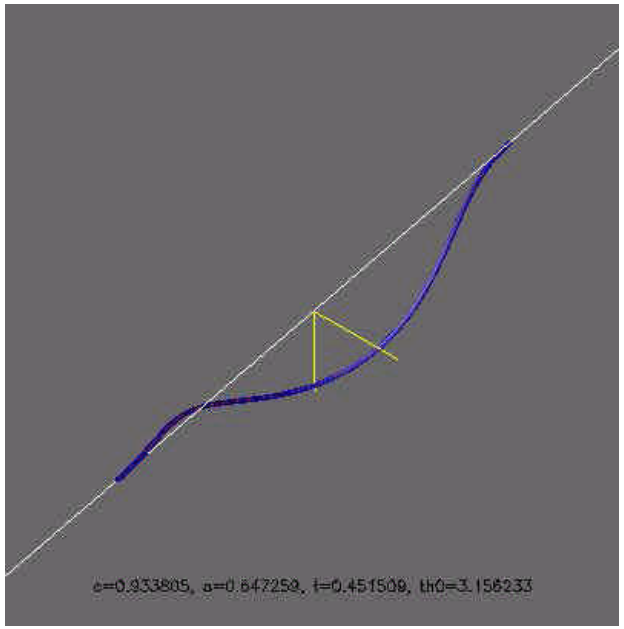


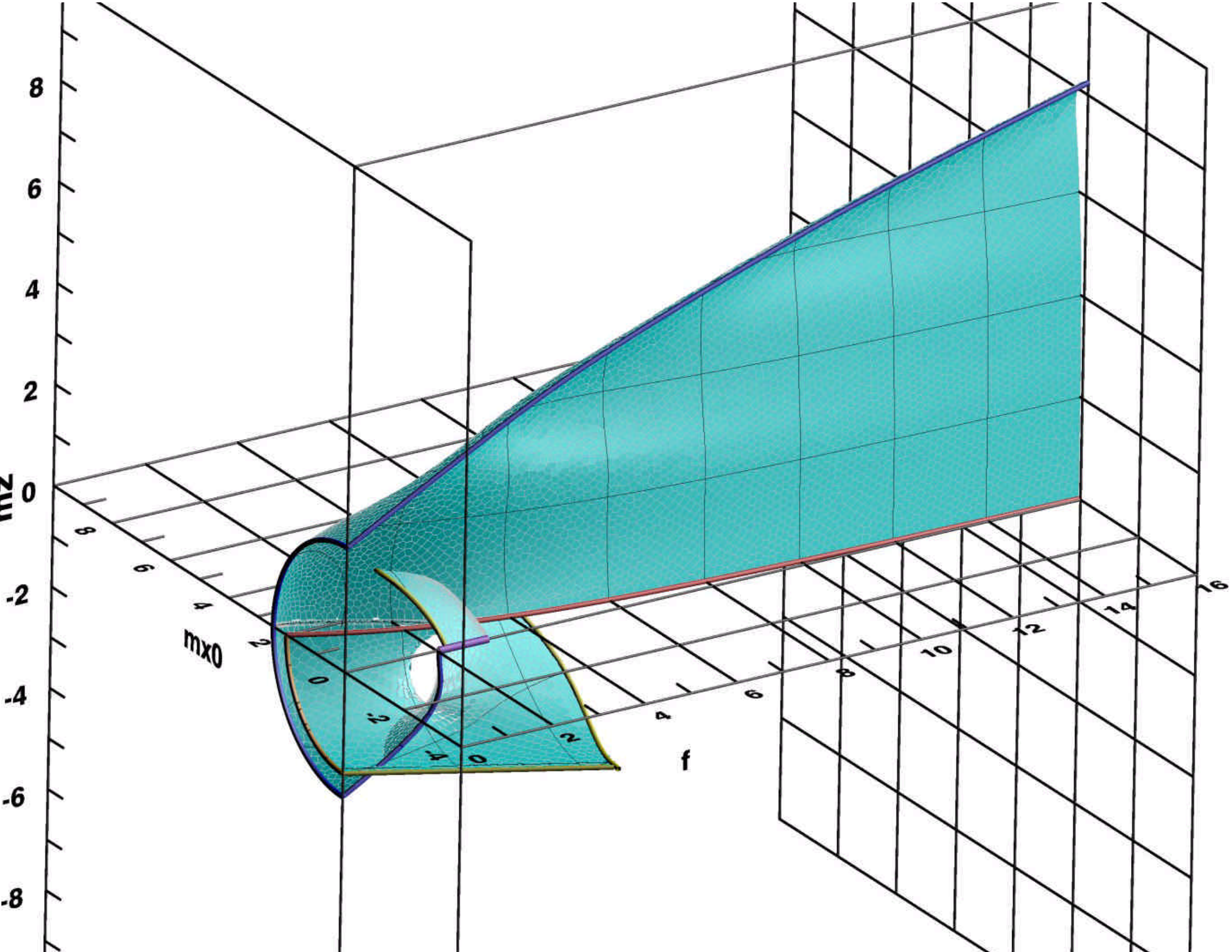


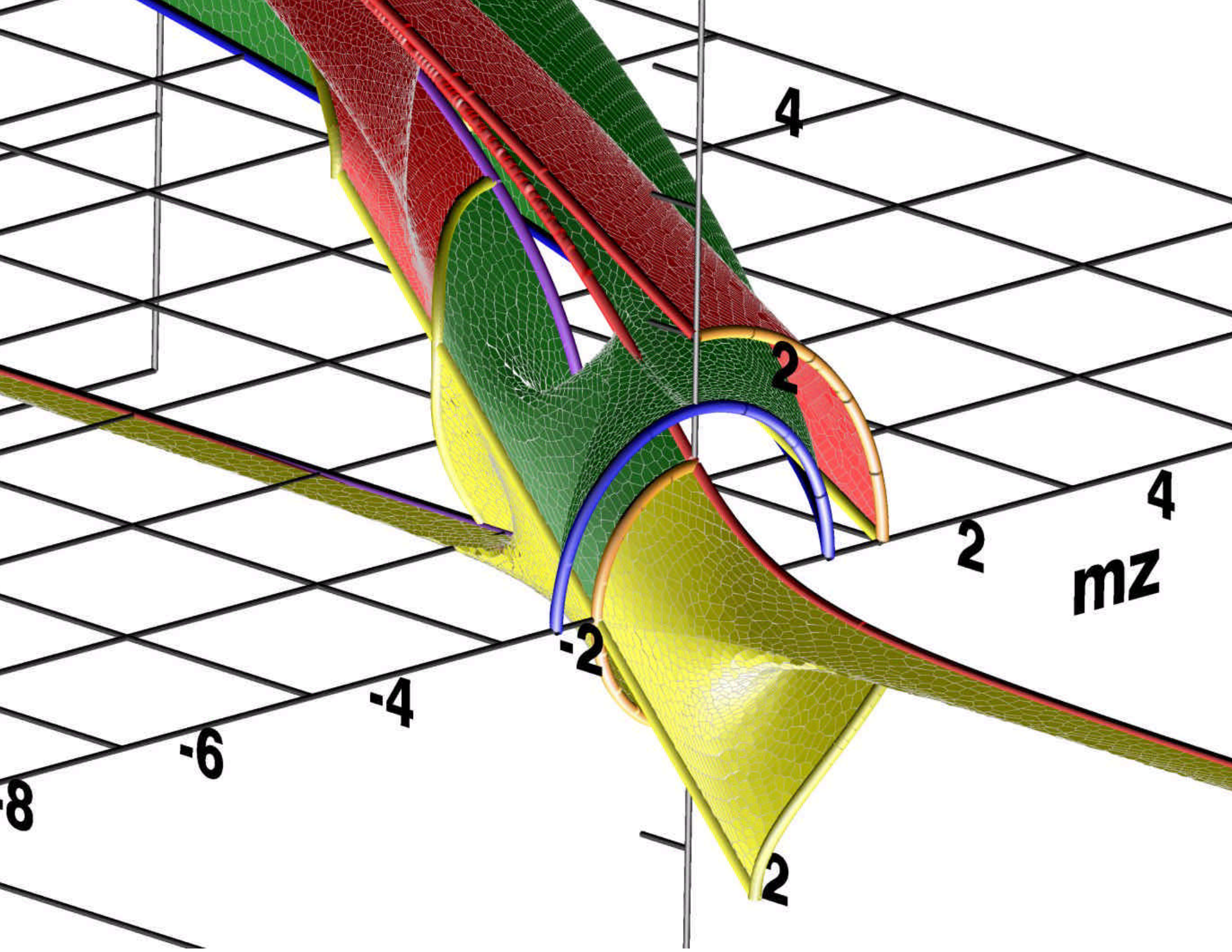






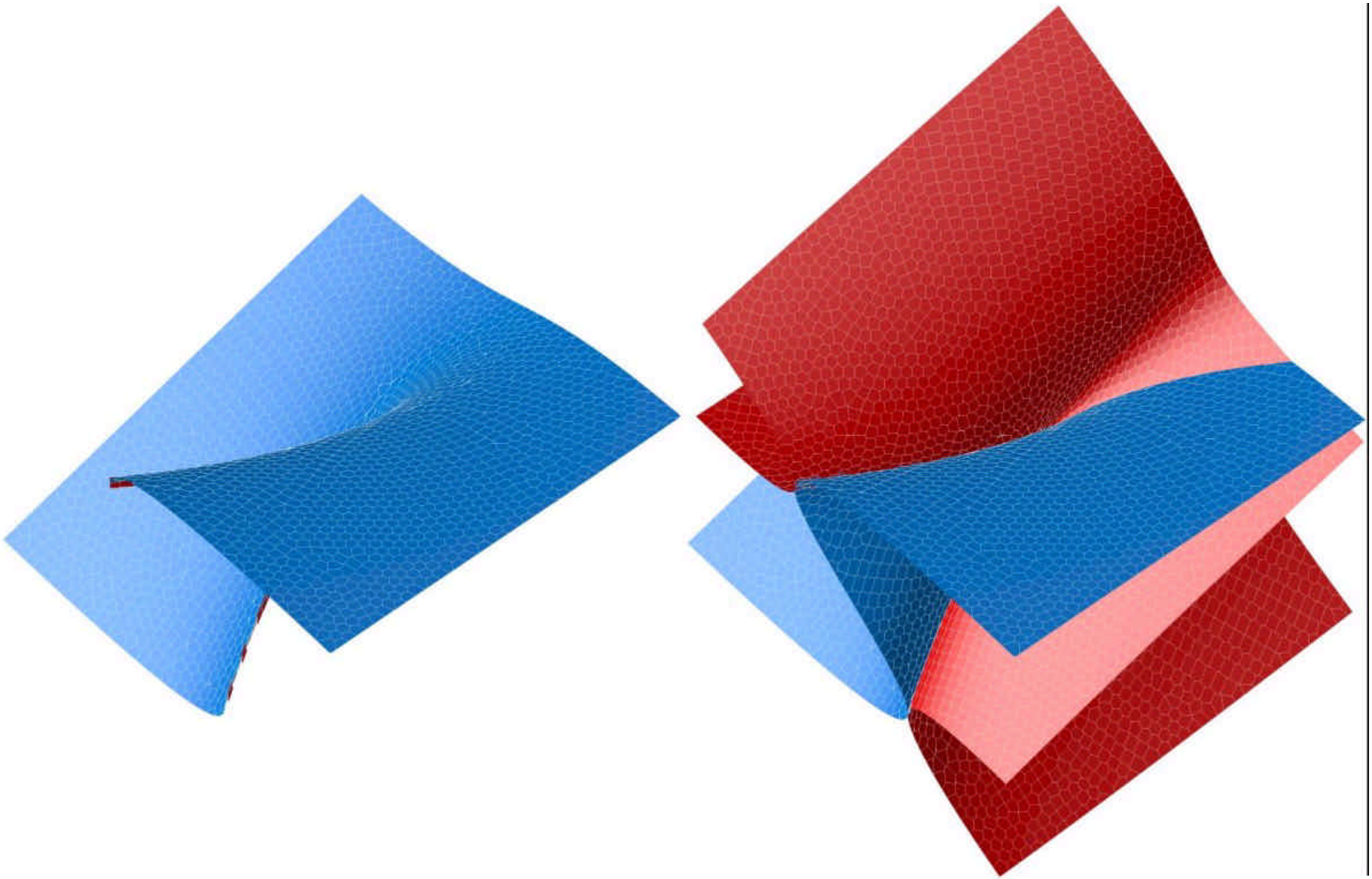



8
6
4
2
0
-2
-4
-6
-8
mz
mx0
f


4
2
4
2
mz
-2
-4
-6
-8
2

Complex Cusp

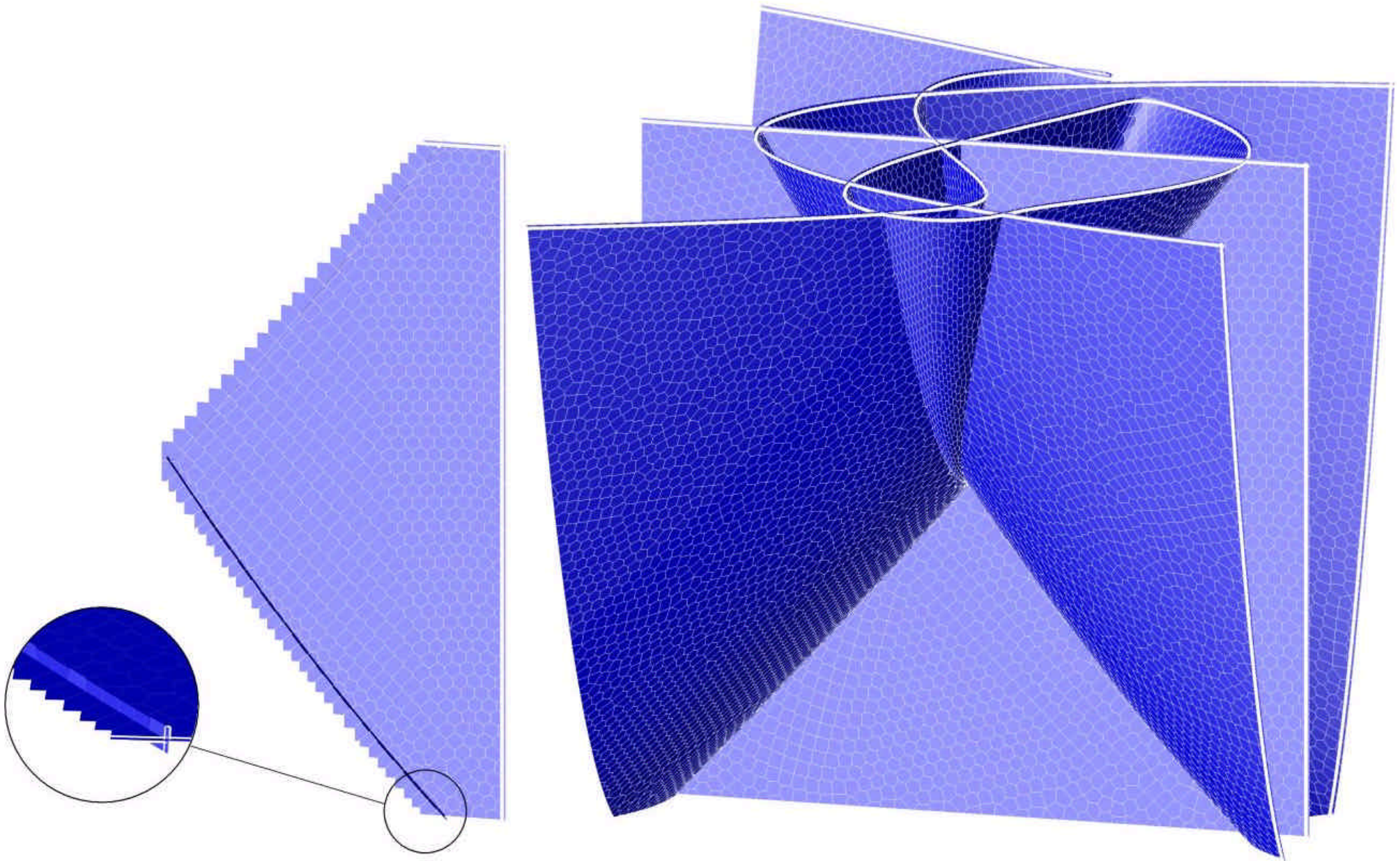
n=4, k=2



$u(u^2 - \mathbf{l}) = \mathbf{m}$

Model of (2,4) mode interaction
in Taylor-Couette Flow

n=4, k=2



Summary - compute a *k*-dim "surface" M

Covers M with a set of well distributed points/polyhedra.

Adds points (neighborhoods) from boundary.

Finds boundary in terms of polyhedra P (like Voronoi polyhedra).

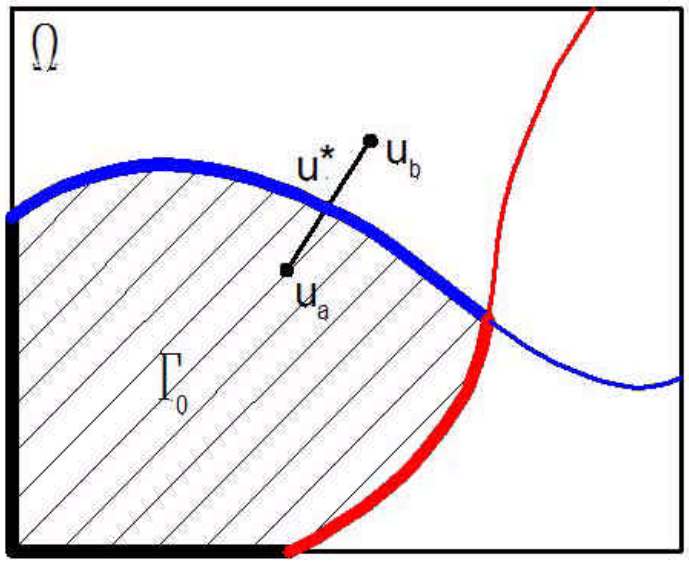
Works in any dimension (subtract half space from P).

Parallel search branch switching.

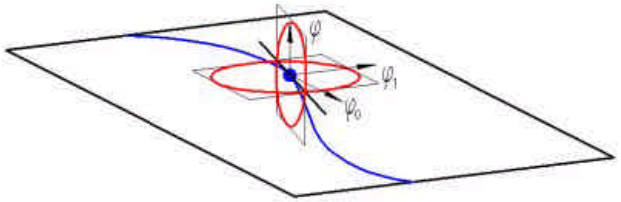
www.coin-or.org/multifario (Open Source)

LOCA

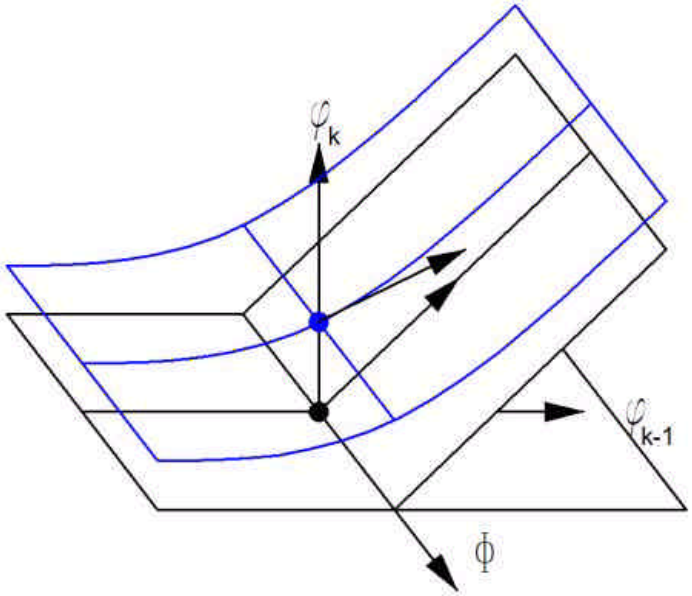
Branch Switching



Detection



Parallel search



Find tangent space of perturbed problem
Common subspace is tangent to sing. crv.