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***Large-Signal Frequency Domain  
Coupled Device and Circuit  
Simulation***

***Karti Mayaram***

*School of EECS  
Oregon State University  
Corvallis, OR 97331*

# Outline

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- **Introduction**
- **Coupled device/circuit simulation**
- **Quasi-static, non-quasi-static, and modified Volterra series harmonic balance analyses**
- **Examples and performance comparison**
- **Conclusions**

# The Modeling Hierarchy

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Speed



Accuracy

High-level  
models

Lumped-  
element  
models

Compact  
models

Numerical  
models

**VHDL-AMS**

**RLC**

**BSIM3**

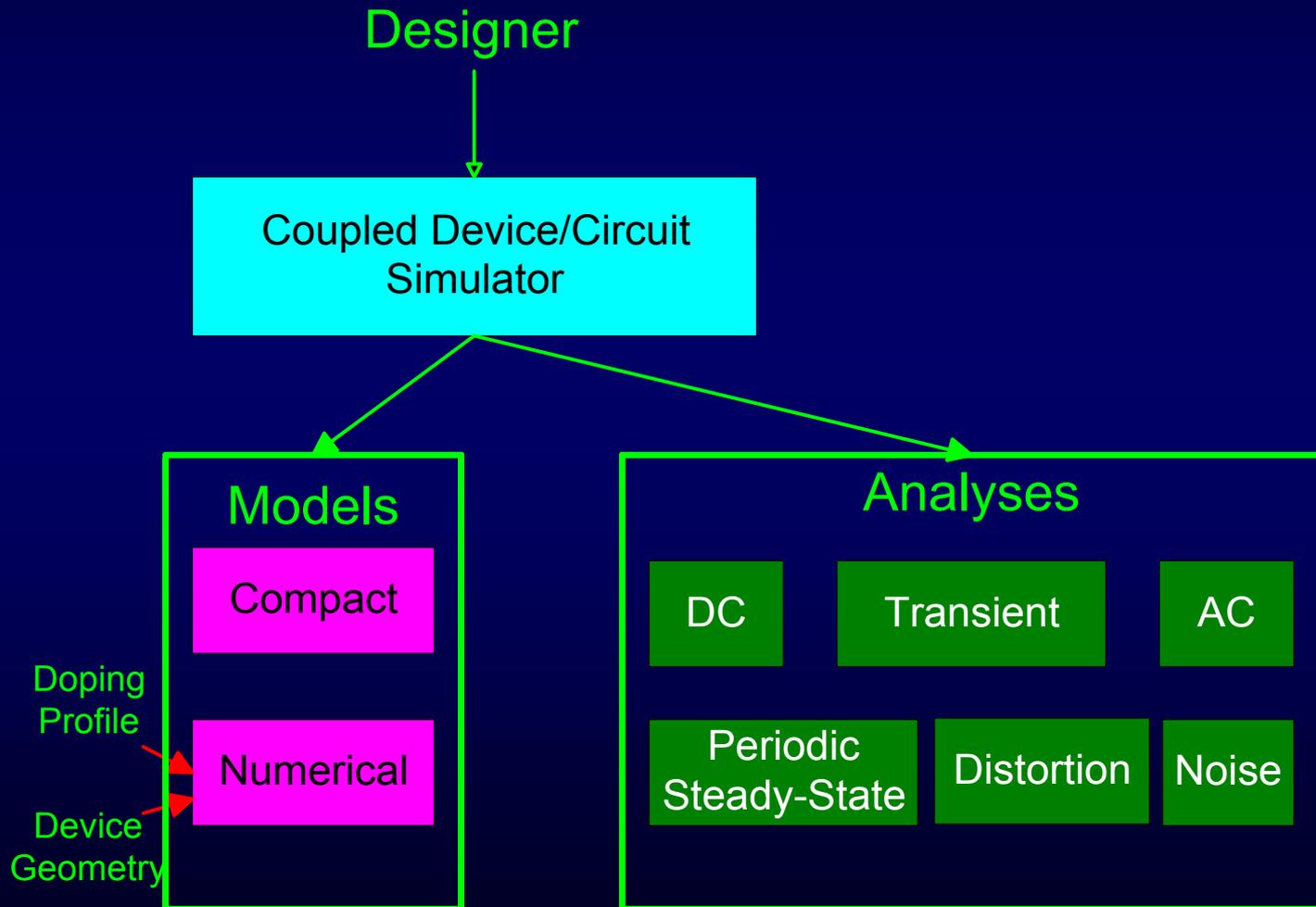
**PISCES**

# Circuit/Device Simulation

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- **Circuit simulation**
  - Analytical (compact) models used: inaccurate under certain conditions
  - + Simulation of multiple devices in a circuit
- **Device simulation**
  - + Based on device physics: accurate
  - Simulation of a single device, no circuit embedding
- **Coupled circuit/device simulation**
  - + Accurate
  - + Simulation of multiple devices

# Coupled Device/Circuit Simulation



# Numerical Model Equations

$$\begin{aligned}\nabla \cdot \epsilon E &= q(N_D - N_A + p - n) & \text{where} \\ \frac{1}{q} \nabla \cdot J_n &= \frac{\partial n}{\partial t} - (G - R) & E = -\nabla \psi \\ \frac{1}{q} \nabla \cdot J_p &= -\frac{\partial p}{\partial t} + (G - R) & J_n = -q\mu_n n \nabla \psi + qD_n \nabla n \\ & & J_p = -q\mu_p p \nabla \psi - qD_p \nabla p\end{aligned}$$

## Device terminal current and voltage

$$I_{device} = \int J_{n,p} \text{ at contact} + \frac{d}{dt} \int \epsilon E \text{ at contact}$$

$$\psi \text{ at contact} = f(V_{device})$$

# Advantages

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- **Simulate critical devices at the device level within a circuit**
  - Solve partial differential equations describing devices coupled to a circuit simulator
- **Predict performance of circuits in absence of compact models for devices**
- **Evaluate influence of process variations on circuit performance**

# Application Examples

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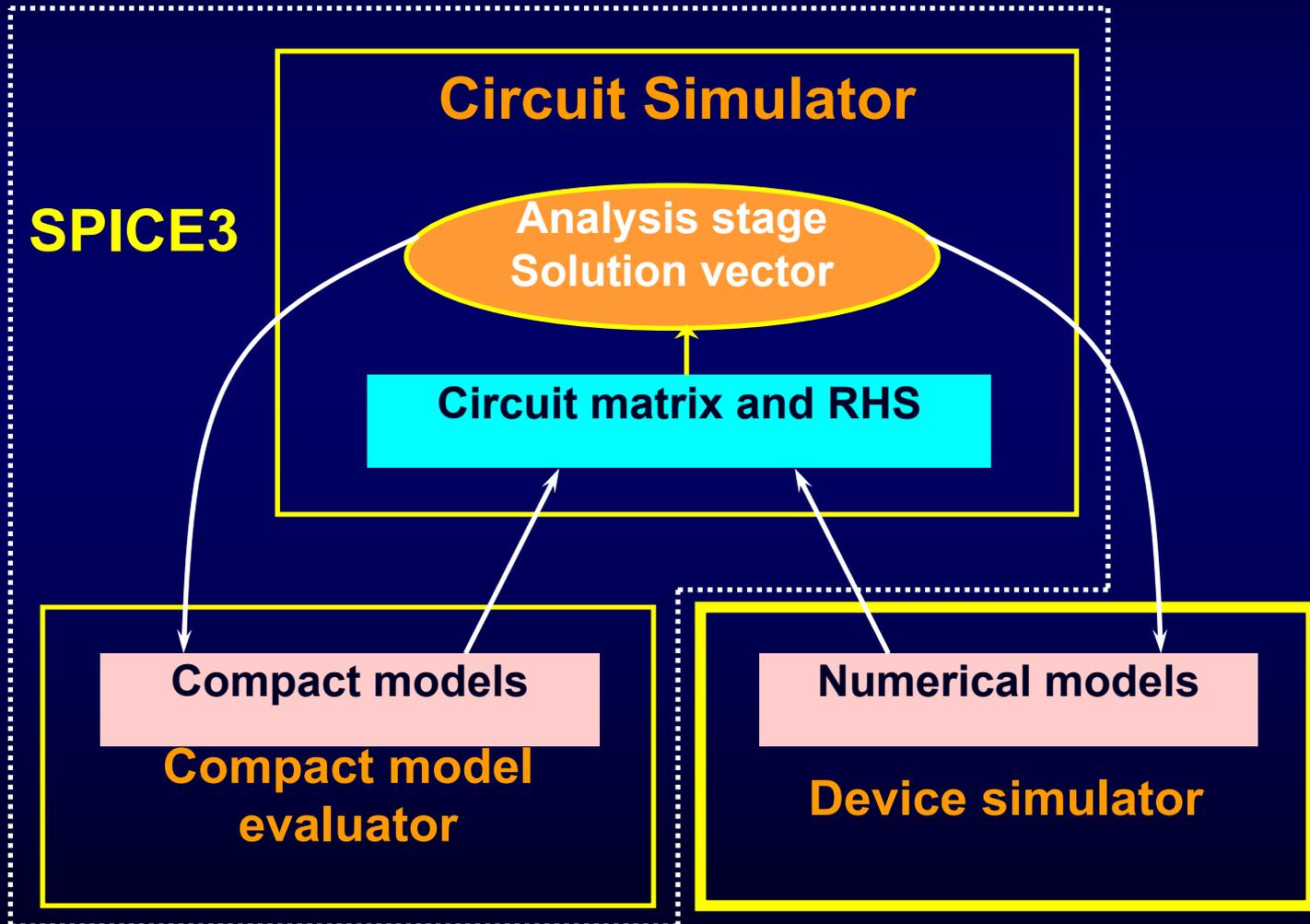
- **Delay analysis of BiCMOS driver circuits**
- **Simulation of power devices**
- **Determination of switch-induced error in MOS switched-capacitor circuits**
- **Simulation of RF circuits**
- **Simulation of single-event-upset in SRAMs**
- **Validation of analytical models**

# Coupled Device and Circuit Simulator (CODECS)

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- **Device-level simulator (PDE solver)**
  - Poisson's and current-continuity equations
  - Accurate terminal conductances and capacitances provided to circuit-level simulator
- **Circuit-level simulator (SPICE3)**
  - Compact model evaluation
  - Simulation engine

# Architecture of CODECS



# RF Circuit Simulation

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- **Accurate and efficient steady-state simulation of RF ICs required for**
  - Distortion, power, frequency, and noise
  - Gain and impedance characteristics
- **Simulation techniques**
  - Time-domain shooting method
  - Harmonic-balance method

# RF Circuit Simulation Issues

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- **Distributed effects in devices important for RF applications**
  - Use physical models in absence of accurate compact models
    - ⇒ Coupled device and circuit simulation for RF circuits

# Harmonic Balance Method

- Truncated Fourier series approximation of  $x(t)$

$$x(t) \approx a_0 + \sum_{i=1}^s (a_i \cos(\omega_i t) + b_i \sin(\omega_i t))$$

- For  $2s+1$  time samples  $x_0 \dots x_{2s}$

$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{2s} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \cos(\omega_1 t_0) & \cdots & \sin(\omega_s t_0) \\ 1 & \cos(\omega_1 t_1) & \cdots & \sin(\omega_s t_1) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \cos(\omega_1 t_{2s}) & \cdots & \sin(\omega_s t_{2s}) \end{bmatrix}}_{\Gamma^{-1}} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ b_s \end{bmatrix}}_{\mathbf{X}} = \Gamma^{-1} \mathbf{X}$$

# Harmonic Balance Method

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- Circuit equations in time domain:

$$i(x(t)) + \frac{d}{dt} q(x(t)) + s(t) = 0$$

- $x(t)$  the vector of circuit waveforms
- $i$  is a vector of contributions from nonreactive elements
- $q$  is a vector of contributions from reactive elements
- $s$  stimulus vector

# Harmonic Balance Method

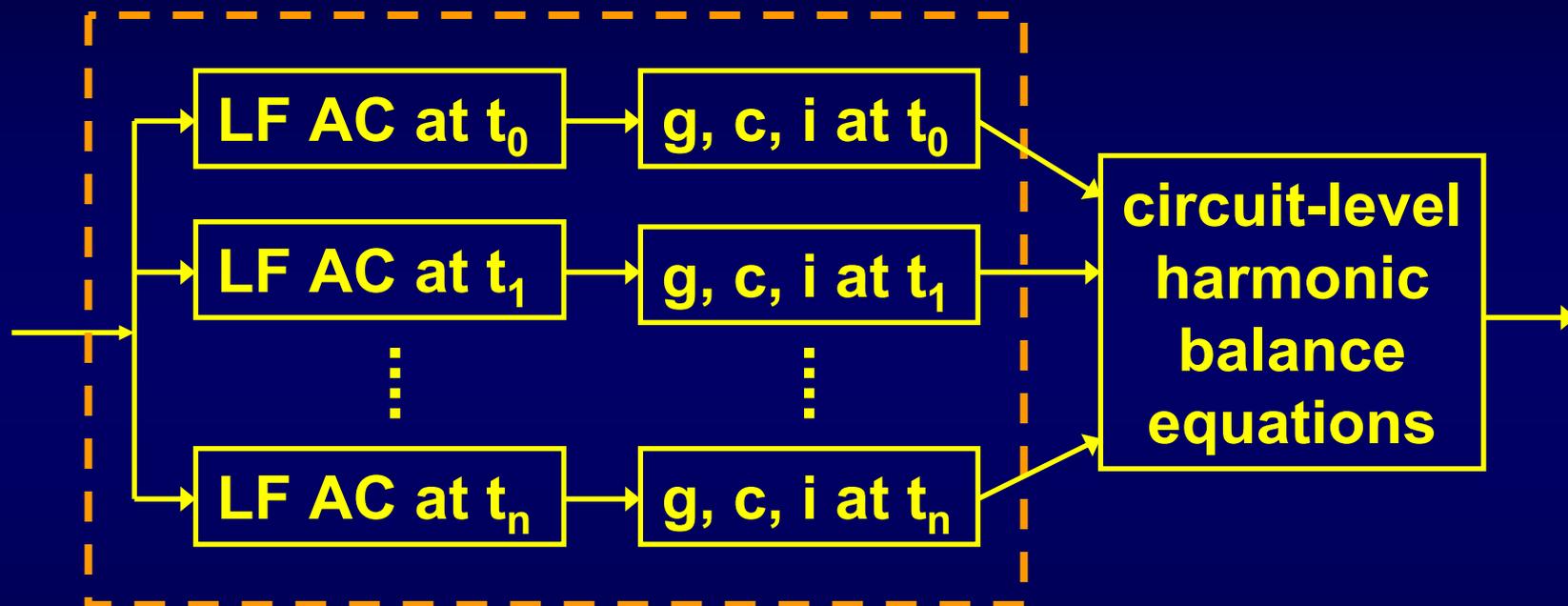
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- Circuit equations in frequency domain:

$$\Gamma i(\Gamma^{-1}X) + \Omega \Gamma q(\Gamma^{-1}X) + S = 0$$

- $X$  the Fourier coefficients of unknown waveforms
- $\Gamma$  and  $\Gamma^{-1}$  the discrete Fourier transform pair
- $i$  time-domain non-reactive elements
- $q$  time-domain reactive elements
- $S$  frequency-domain stimulus
- $\Omega$  frequency-domain representation of time derivative

# Quasi-Static Harmonic Balance



- **Harmonic balance at circuit level**
- **Quasi-static approach**
  - g and c based on static solutions of devices

# Quasi-Static Harmonic Balance

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- **Efficient**
  - Circuit waveforms as unknowns
  - Small harmonic balance matrix
  - No modification to device simulator
- **Accuracy problems**
  - When terminal voltages vary faster than device transit times
  - $g$  and  $c$  are frequency dependent

⇒ **Non-quasi-static harmonic balance analysis**

# Non-Quasi-Static (NQS) Harmonic Balance

- Solve the complete system of equations at both circuit and device levels

$$\left\{ \begin{array}{l} f_d = d(w, v) + \frac{d}{dt}(a \cdot w) = 0 \\ f_c = \underbrace{l(w, v) + \frac{d}{dt}D(w, v)}_{i_d} + i(v) + \frac{d}{dt}q(v) + s = 0 \end{array} \right.$$

- $f_d$  and  $f_c$  device- and circuit-level equations
- $w$  and  $v$  device- and circuit-level unknowns
- $a$  is a constant  $\in \{+1, -1, 0\}$
- $i_d$  currents from numerical devices
- $l$  and  $D$  conduction and displacement terms

# NQS Harmonic Balance Equations

- Solve using Newton's method

$$\left( \begin{bmatrix} \mathbf{J}_w & \mathbf{J}_v \\ \mathbf{G}_w^I & \mathbf{G}_v^I + \mathbf{G} \end{bmatrix} + \mathbf{T} \begin{bmatrix} \mathbf{a} & \mathbf{0} \\ \mathbf{C}_w^D & \mathbf{C}_v^D + \mathbf{C} \end{bmatrix} \right) \begin{bmatrix} \Delta \mathbf{w} \\ \Delta \mathbf{v} \end{bmatrix} = - \begin{bmatrix} \mathbf{f}_d \\ \mathbf{f}_c \end{bmatrix}$$

–  $\mathbf{T} = \Gamma^{-1} \Omega \Gamma = \frac{d}{dt}$  in harmonic balance method

–  $\mathbf{J}_w, \mathbf{J}_v, \mathbf{G}_w^I, \mathbf{G}_v^I, \mathbf{C}_w^D, \mathbf{C}_v^D, \mathbf{G}$ , and  $\mathbf{C}$  are corresponding derivatives to device- and circuit-level unknowns

# NQS Harmonic Balance Solution

- Eliminate  $\Delta w$

$$\Rightarrow \Delta w = (\mathbf{J}_w + \mathbf{T}a)^{-1}(-\mathbf{f}_d - \mathbf{J}_v \Delta v)$$

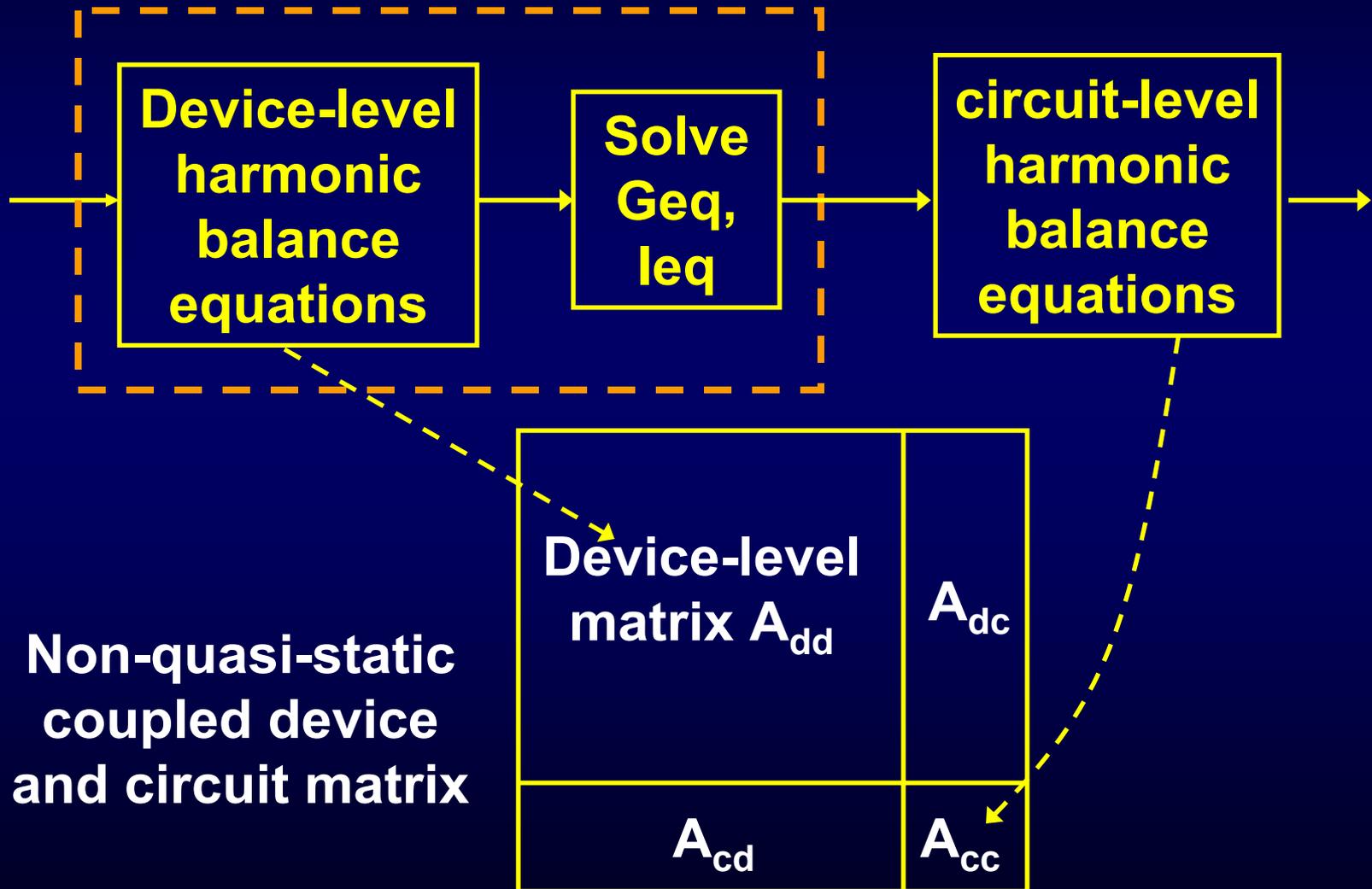
$$\left[ \mathbf{G}_{eq} + (\mathbf{G} + \mathbf{T}C) \right] \Delta v = -\mathbf{f}_c - \mathbf{I}_{eq}$$

where,  $\mathbf{G}_{eq} = (\mathbf{G}_w^I + \mathbf{T}C_w^D)(\mathbf{J}_w + \mathbf{T}a)^{-1}(-\mathbf{J}_v) + (\mathbf{G}_v^I + \mathbf{T}C_v^D)$

$$\mathbf{I}_{eq} = (\mathbf{G}_w^I + \mathbf{T}C_w^D)(\mathbf{J}_w + \mathbf{T}a)^{-1}(-\mathbf{f}_d)$$

- $\mathbf{G}_{eq}$  and  $\mathbf{I}_{eq}$  are the contribution of numerical device to the circuit-level equation

# NQS Harmonic Balance Implementation



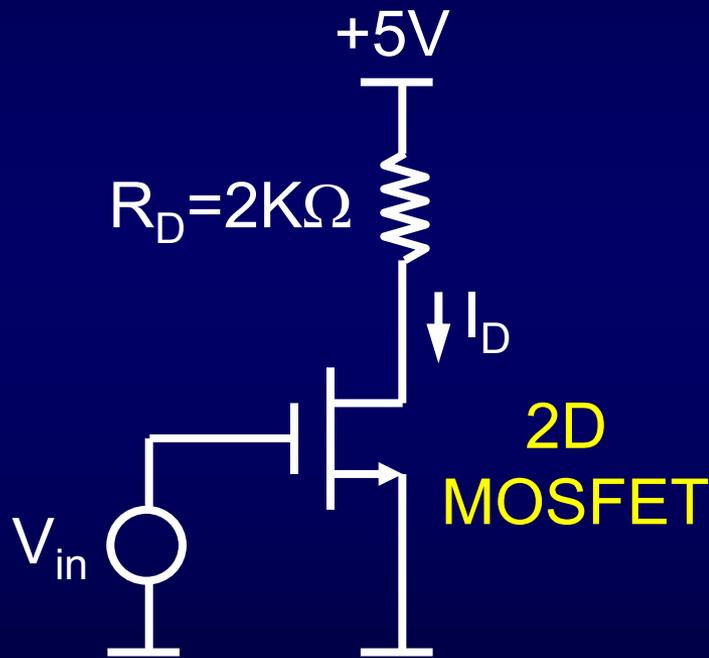
# NQS Harmonic Balance Summary

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- **Accurate for very high frequencies**
  - Harmonic balance at both device- and circuit- levels
  - Internal dynamics of devices taken into account
- **Expensive**
  - A large number of device-level unknowns
  - Large dense system matrix
  - Need extensive modifications to device simulator

# Comparison of NQS and QS

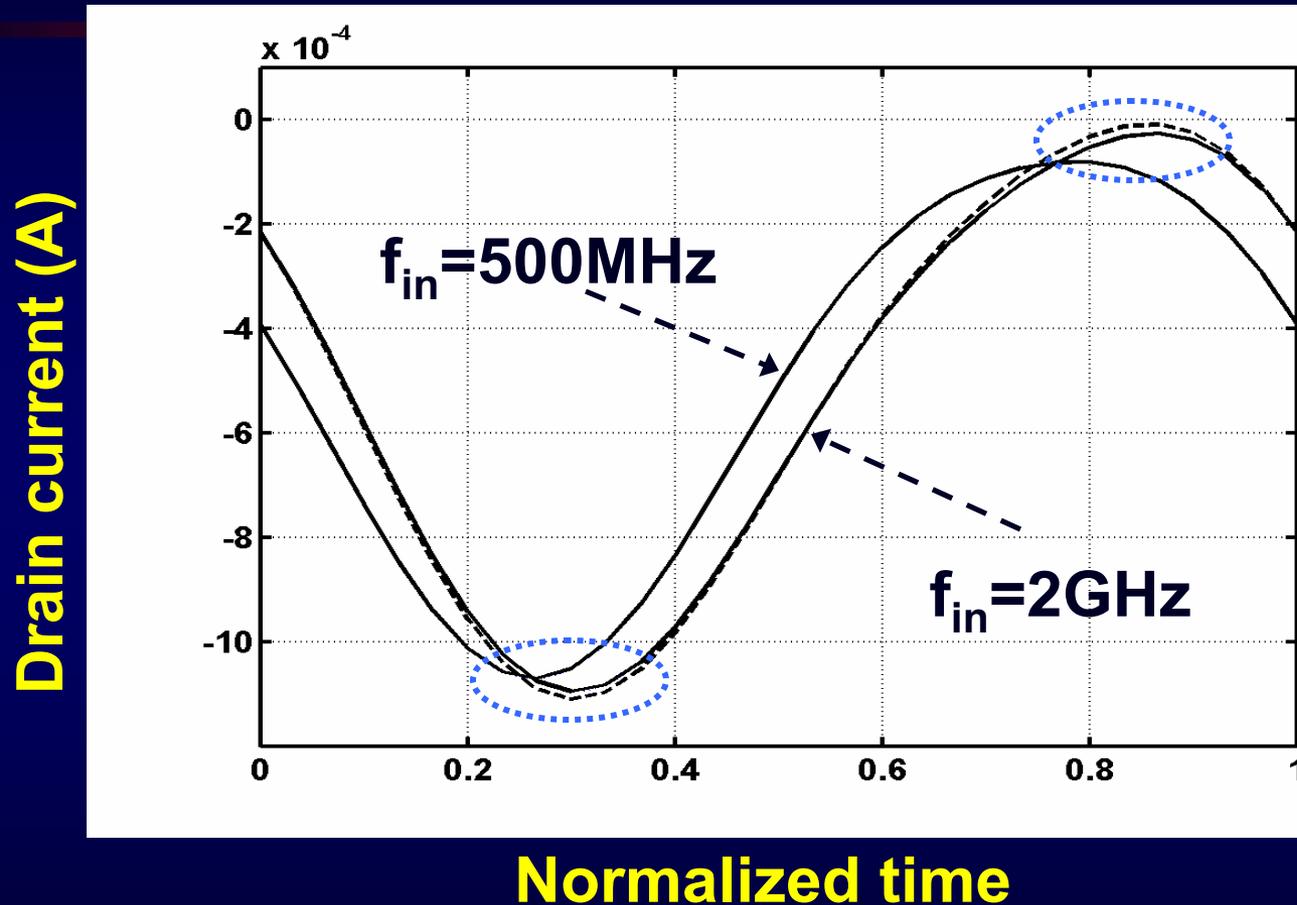
- Simple CS amplifier



$$V_{in} = 2 + 0.5 \sin(2\pi \cdot f_{in} \cdot t)$$

- $f_{in} = 500\text{MHz}$   
 $2\text{GHz}$   
 $5\text{GHz}$
- 2D MOSFET
  - Mesh points:  
 $31 \times 19 = 589$
  - $f_T = 2\text{GHz}$

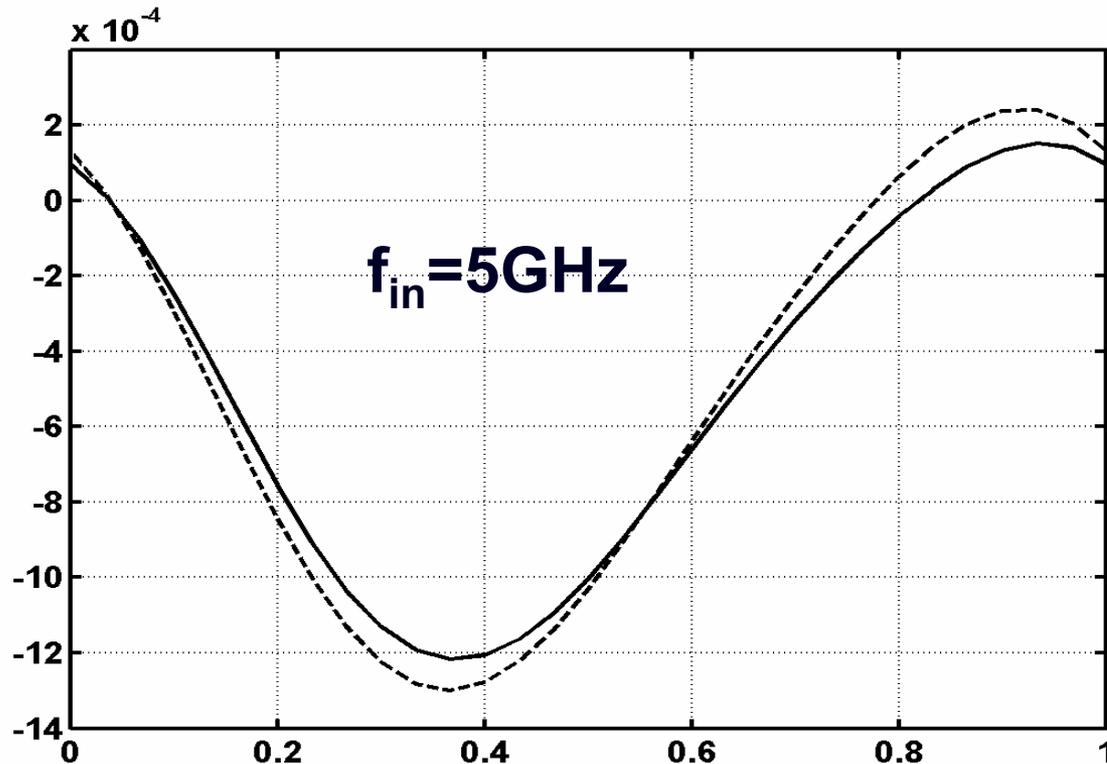
# Comparison of Accuracy (CS Amp)



- ..... Transient
- Quasi-static harmonic balance (max error 3.8%)
- Non-quasi-static harmonic balance

# Comparison of Accuracy (CS Amp)

Drain current (A)



Normalized time

- ..... Transient
- Quasi-static harmonic balance (max error 16.8%)
- Non-quasi-static harmonic balance

# Comparison of Simulation Complexity (CS Amp)

	Quasi-static method	Non-quasi-static method
No. freqs chosen	4	4
Simulation time	14s	3840s
Memory used	5.3M	103M
No. eqns for HB at circuit level	28	28
No. eqns for HB at device level	0	6447 (7x921)
Largest matrix	921 x 921	6447 x 6447

← 270x

← 20x

# Comparison of Simulation Complexity (SCP)

	Quasi-static method	Non-quasi-static method
No. freqs chosen	4	4
Simulation time	22s	8640s
Memory used	6.9M	203M
No. eqns for HB at circuit level	56	56
No. eqns for HB at device level	0	12894 (7x2x921)
Largest matrix	921 x 921	6447 x 6447

← 390x

← 30x

# Comparison of Sparse and UMFPACK

	Sparse	UMFPACK	
No. freqs chosen	4	4	
Simulation time	19650s	3471s	← 5.7x
Memory used	409M	165M	← 2.5x
No. eqns for HB at circuit level	63	63	
No. eqns for HB at device level	18858	18858	
Largest matrix	9429x9429	9429x9429	

# Modified-Volterra-Series (MVS) Device Model

- First order modified Volterra series\*

- Terminal voltage

$$v(t) = \sum_{h=-H}^{h=+H} V_h e^{j2\pi f_h t}$$

- Terminal current

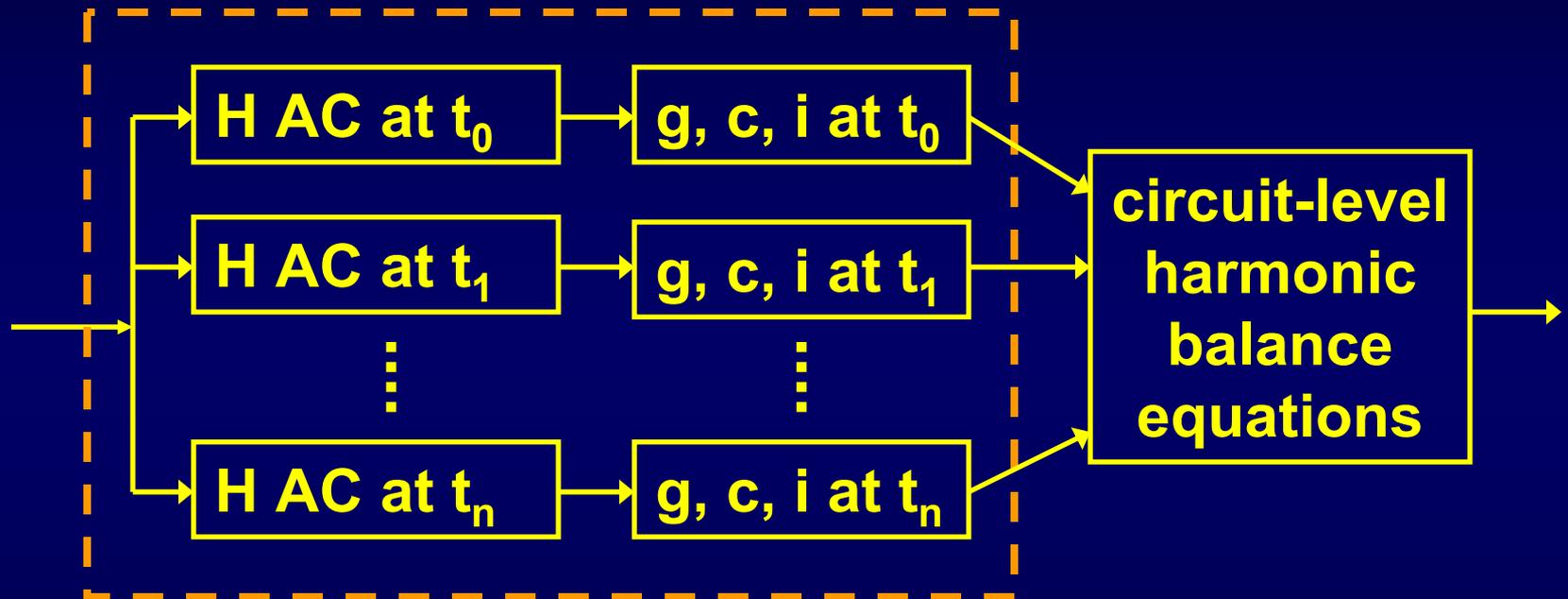
$$i_d = z_0(v) + \sum_{h=-H}^{h=+H} D\{v, f_h\} V_h e^{j2\pi f_h t}$$

Static current

$Y(f_h, v) - Y(0, v)$

\* F. Filicori, G. Vannini. V. A. Monaco, July 1992.

# MVS Harmonic Balance Implementation



- Harmonic balance at circuit level
- H AC analyses at H frequencies (longer time)
- i includes dynamic current

# MVS Harmonic Balance Summary

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- **Efficiency**
  - Circuit waveforms as unknowns
  - No modification to device simulator
  - Longer time for H AC analyses
- **Accuracy**
  - Frequency dependence of  $g$  and  $c$
  - Small-signal internal dynamics
  - Improvement for high frequency and small amplitude signals

# Differences Between Approaches

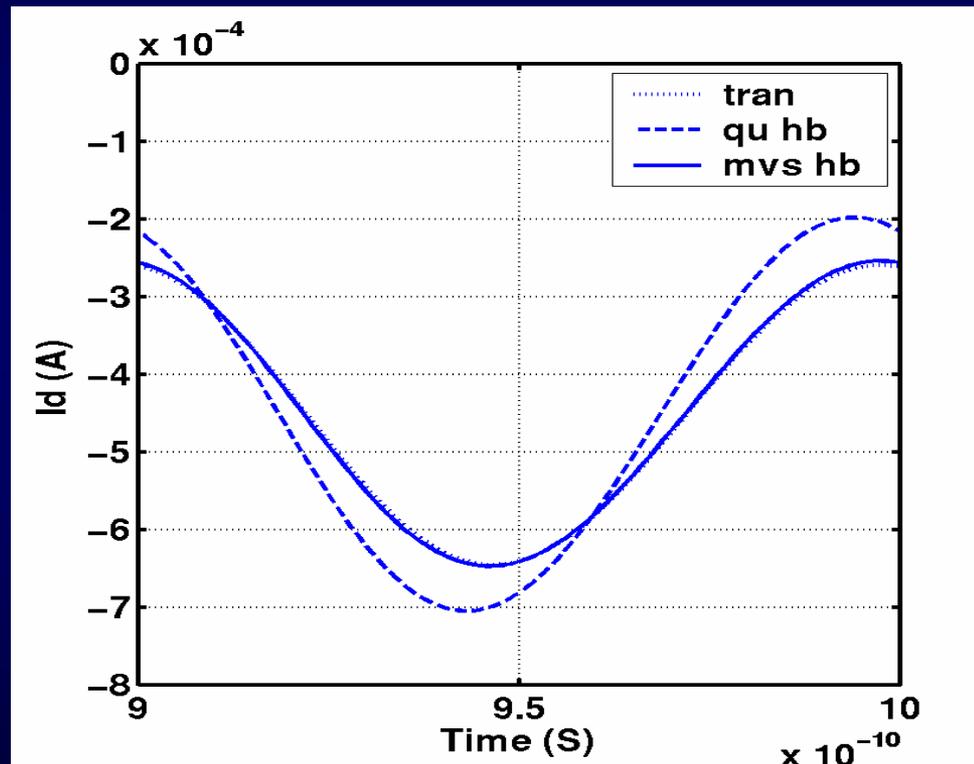
- Handling of numerical device dynamics

	Device terminal dynamics*	Large-signal internal dynamics	Small-signal internal dynamics
NQS	Yes	Yes	Yes
QS	Yes	No	No
MVS	Yes	No	Yes

\* terminal dynamics  $\frac{d}{dt}D(\mathbf{w}, \mathbf{v})$

# Accuracy Comparison of QS and MVS

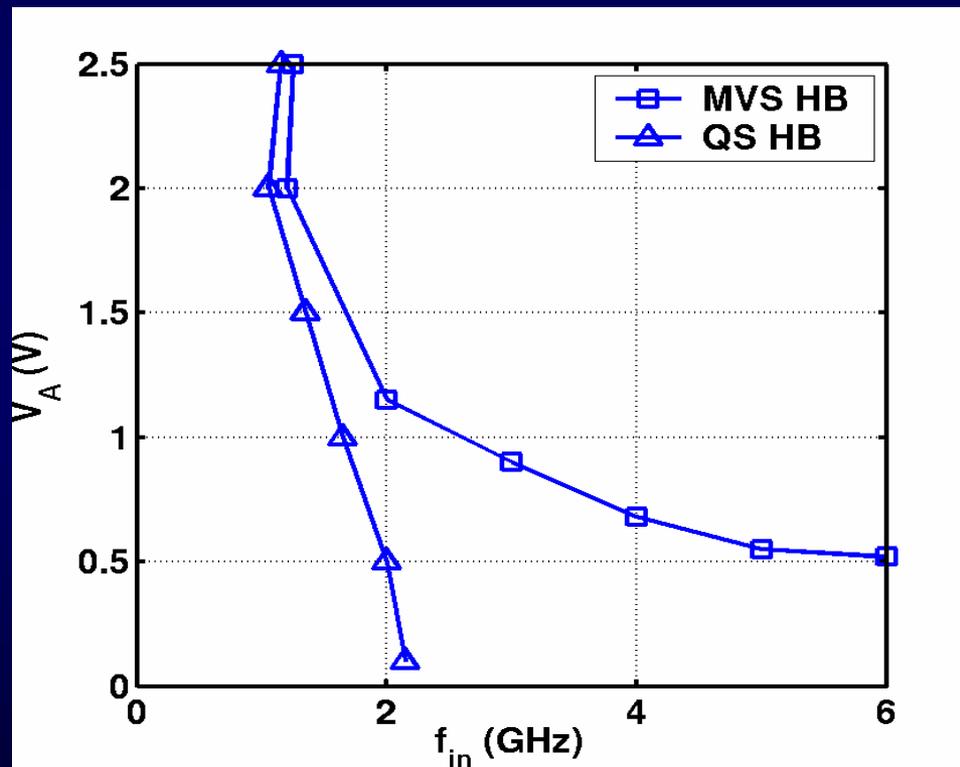
- Simple CS amplifier  $V_{in} = 2 + 0.1 \sin(2\pi \cdot 10 \text{GHz} \cdot t)$



- Accuracy improved with MVS HB
- Phase and magnitude errors from QS corrected

# Constant Error (5%) Loci of QS and MVS

- $V_{in} = 2 + V_A \sin(2\pi \cdot f_{in} \cdot t)$



- Accuracy of MVS HB method depends on  $V_A \cdot f_{in}$
- Accuracy of QS HB mainly depends on  $f_{in}$

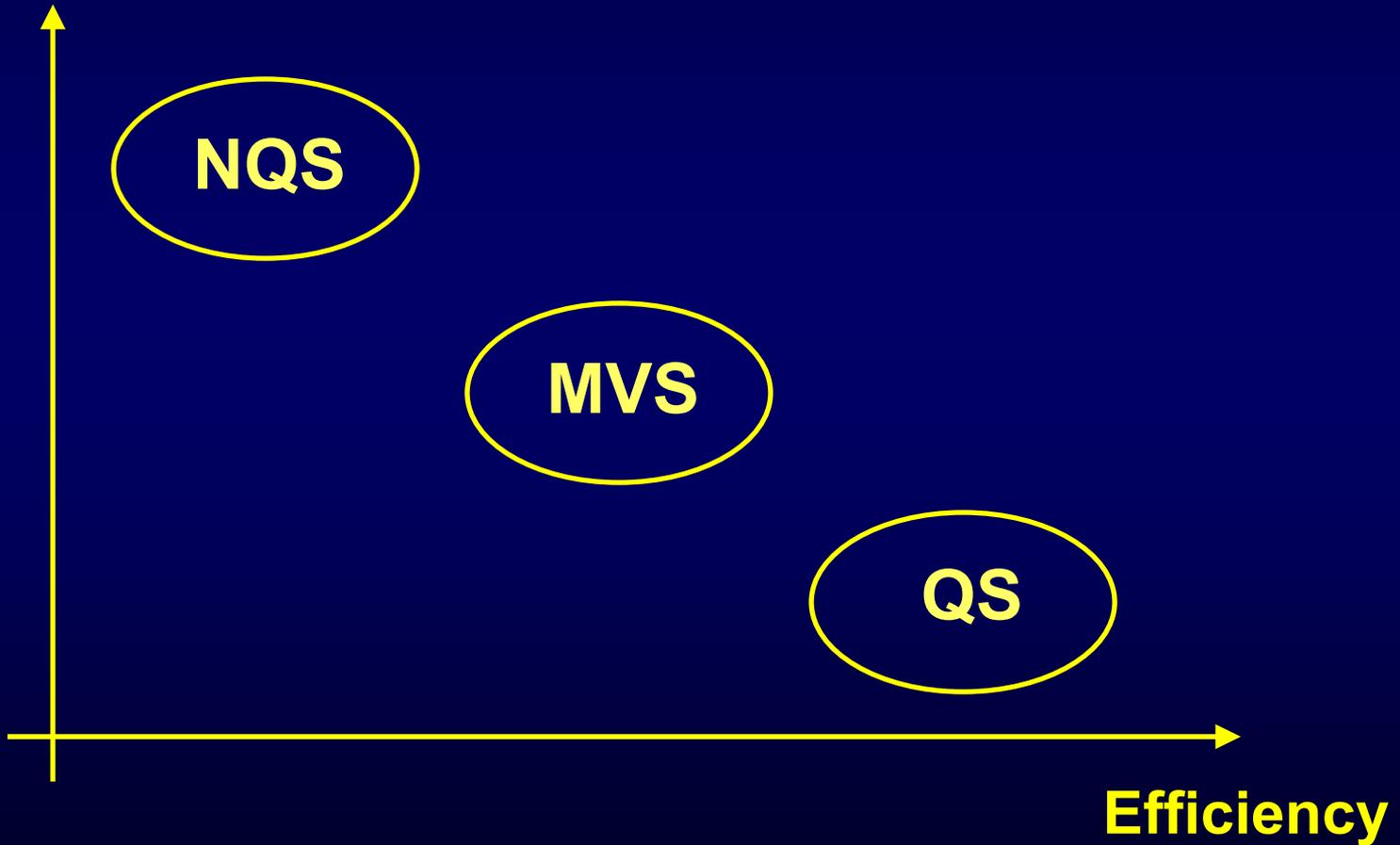
# Summary of Harmonic Balance Implementation

	NQS method	MVS method	QS method
HB level	Circuit & device	Circuit	Circuit
Modifications to device simulator	Extensive	None	None
Memory used	Large	Small	Small
Simulation time	Slow	Fair	Fast
Accuracy	Excellent	Improved for small signals	Limited by $f_T$

# Tradeoff Between Approaches

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Accuracy



# Conclusions

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- **Coupled circuit/device simulations for accurate simulation of circuits**
- **Provide a direct link between technology changes and circuit performance**
- **Frequency-domain harmonic balance method for RF applications**
  - Three different implementation approaches
  - Tradeoff between accuracy and efficiency
- **Need faster solution methods for PDEs**

# Backup

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# Static, Quasi-Static, and Non-Quasi-Static Models

- **Non-quasi-static (NQS) model:**

$$d(w, v) + \frac{d}{dt}(a \cdot w) = 0 \quad I_d = I_{cond}(w, v) + \frac{d}{dt} D_{displ}(w, v)$$

- **Quasi-static (QS) model:**

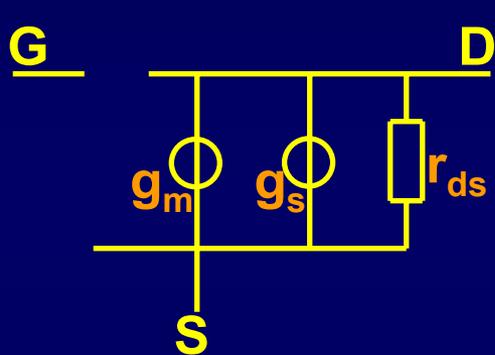
$$\cancel{d(w, v) + \frac{d}{dt}(a \cdot w) = 0} \quad \cancel{I_d = I_{cond}(w, v) + \frac{d}{dt} D_{displ}(w, v)}$$

- **Static model:**

$$\cancel{d(w, v) + \frac{d}{dt}(a \cdot w) = 0} \quad \cancel{I_d = I_{cond}(w, v) + \frac{d}{dt} D_{displ}(w, v)}$$

# Static, QS, and NQS models (small signal)

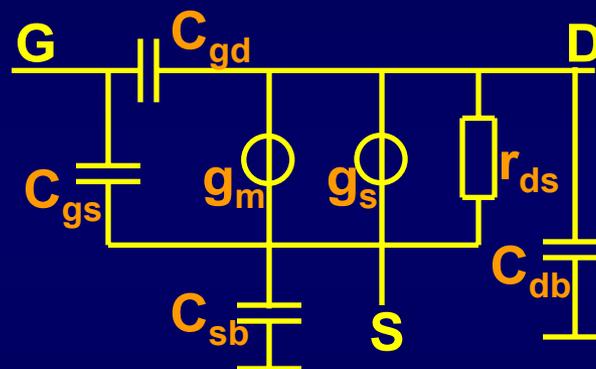
## • MOSFET small-signal model



Static model:

$$C=0$$

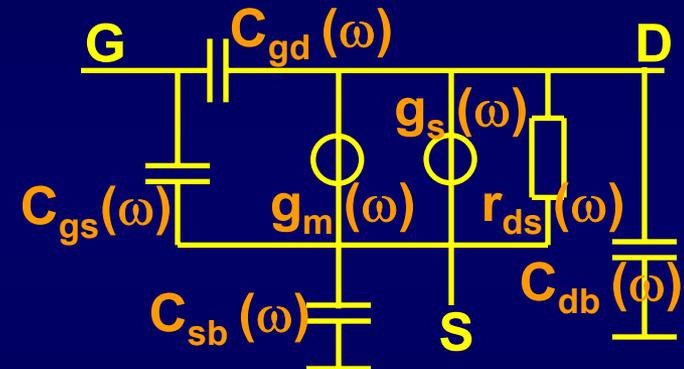
$$G(V_{\text{bias}})$$



QS model:

$$C(V_{\text{bias}})$$

$$G(V_{\text{bias}})$$



NQS model:

$$C(V_{\text{bias}}, \omega)$$

$$G(V_{\text{bias}}, \omega)$$