Numerical Methods for Partial Differential Equations (Math 578)

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1 General Description

Numerical simulation has reached certain levels of maturity throughout science and engineering and as computer resources become more readily available, this technology is transitioning from "advanced capabilities" to the "default analysis tool" that should be part of every scientist/mathematician/engineer's toolbox. This course is designed to 1) build up the theoretical foundation, 2) practise computational implementation, and 3) work with practical applications. The goal is to provide students with a skill set from which simulation can be used to investigate more complex dynamics and advanced concepts.

2 Course Outline

- Introduction - finite difference discretization, boundary conditions, truncation errors, stability, consistency.
- Elliptic problems - classification, ordering, accuracy
- Iterative methods - Gauss-Seidel, conjugate gradient, preconditioning, GMRES, multigrid
- Initial value problems - linear ordinary differential equations, Lipschitz continuity, time integration.
- Initial value problems - stability, convergence, non-linear problems, stiff ODEs,
- Parabolic and hyperbolic problems - methods of lines, Von Neumann analysis, multi-dimensional problems, Lax-Wendroff, Courant-Friedrichs-Lewy condition
- Advanced concepts - mixed equations, finite element discretization, production finite element simulation tools, adjoint formulation, optimization.

3 Projects, Assignments, Format

Numerical methods for PDEs requires an understanding of algorithms, implementation, and problem solving. To meet those goals, approximately one to two week projects will be assigned consisting of algorithm implementation and solving specific problems. At the conclusion of the class, students will have developed a comprehensive collection of numerical methods and will have solved a variety of problems. These assignments will consist of implementation of algorithms that have been discussed during class lectures. The coding language will be Matlab and some basic C++ (not a prerequisite) for the finite element simulation tools.

4 Schedule, Prerequisites,Grading, and Texts

The class is scheduled for the Spring of 2010 semester (MW 5-6:15 pm). Recommended prerequisites for the class consist of calculus, basic linear algebra, an introduction to partial differential equation (math 463),and numerical analysis (math 504). The recommended text is Finite Difference Methods for Ordinary and Partial Differential Equations" by Randall J. Leveque.