Choosing Corners of Rectangles
for Mapped Meshing

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Motivation

Structured quadrilateral meshing

Quads

• better analysis for structural mechanics,
  • but, very dependent on shape of quads (not topology)

Structured

• lower memory, faster speed than unstructured
  • In 3d, many hex algorithms 2.5 dimensional
  • Applications: car crankshafts, tire treads & cross-sections, weapon components

+ twists, turns, and branching
Problem Definition

Complication: some intervals fixed
Problem definition

1. Precise

Given a surface with prescribed (hard-set) and desired (soft-set) intervals on its curves, for which mapped-meshing is suitable,

choose four vertices as corners, such that a good quality mapped-mesh results (feasible).

2. Abstract

Given a surface that resembles a rectangle,

choose which curves of the surface are on opposite sides.

Related: Is mapped-meshing appropriate for a given surface?
Previous results

Metric to determine closeness between two shapes [Arkin et al].

- For all matchings of vertices, angles
  - integrate distance\(^2\) between \(\Theta(s)\)'s,
  - take smallest integral as metric
Previous results

Related ideas

Hausdorff metric...

Smallest enclosing rectangle...

Medial axis, skeleton...

Our problem

• Indeterminate rectangle height & length

• Input surface curve lengths non-Euclidean
  • some fixed, some flexible. User, previous mesh.

• angles at corners very important
  • curves could be curved, treat as straight
Corner Function

3 Goals (angle & aspect ratio of elements)

- angles at corners small ($\pi/2$), bound away from $\pi$
  - $\text{corner\_angle}^{1.7}$

- opposite sides’ intervals equal, relative change small
  - $\text{opp\_ratio}^{1.7} + \text{new/current}$

- no turning between corners, square v.s. elongated
  - $\text{turn}^{1.7}$

$v.s.$ 
$a^{1.7} + b^{1.7}$

$(a+b)^{1.7}$

$a$ 
$b$
Corner function

For a given choice of corners, compute “corner function”

• Gives single value to desirability of given corners.
• 3 goals weighted through practice, examples.
• Doesn’t consider side angles (is a rectangle?)

Heuristic: try all possible combinations of corners. Choose smallest value.

• $O(n^4)$, $n = \# (\text{convex})$ vertices.
  • Shift corner & update value, $O(1)$
• Done for $n<9$, and for
• Post-curve meshing, corners exactly opposite, $O(m^2)$. 
Faster Heuristic

Faster heuristic, $O(n^2)$, for $n>9$

- pick opposite corners, best 2-corner function
  - different goal on turning ($\pi/2$)
  - bisects, consider “best-remaining” corners
- pick remaining 2 corners, best 4-corner function

Occasional failure

- hard-sets -> interval-matching infeasible

If failure (hard-sets)
  - provably get feasible corners (if any)
  - heuristically shift, singly & in pairs
Infeasibility

For one surface, from hard-sets

- provable fallbacks

For collection of surfaces,

- open, NP-complete (Mohring et al)

\[ a = a + b, \quad b = 0, \]
Provable algorithm
one surface

case 3+ soft-set curves, feasible

red = soft, variable > 0
black = hard, fixed

case 2 soft-set curves
case 1 soft-set curve

blue = equal hard-set subsequences

any two pairs that exactly divide
like post-curve meshing not-quite lock -step
but heuristic always finds

case 0 soft-set curves
Examples

Choosing corners after curve-meshing needed.
GoodYear Tire Examples

Cross section

Tread
Examples

- Flat
- Reflex
- Bad, align
Examples, Rectangle & Triangle

Triangle intervals less constrained:

• sum-even, + triangle inequality
Open Problems

Is the polygon close to a rectangle?

- White, add reflex angle considerations

Choosing "corners" for rectilinear shapes

- Vavasis & Driscoll, CRDT
- Long skinny features important. Skeleton?

Chose corners and intervals together

- Mohring et al.
  - NP-complete for some set of primitives
  - Network flow instead of LP formulation
- Volume meshing constraints
Conclusions

Heuristic, $O(n^2)$, hard-sets difficult

Provable algorithms, heuristic shift

Extended to triangles (not immediate)

Problems with coarse meshes of curves

- if $n<4$, assume corner mid-curve,
- else always treat curves as straight