Unstructured Primal-Dual Mesh Improvement and Generation

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Poster Abstract

We investigate the mathematics and algorithms for primal-dual mesh improvement and generation. We made progress on simultaneously-good primal-dual mesh quality, resampling positions for mesh improvement, and dual Voronoi polyhedral mesh generation.

In simultaneously-good primal/dual mesh quality, the degrees of freedom are both node positions and power weights. We analyzed the traditional HOT-energy metric, which bounds the discretization error of the diagonalized Hodge-star operator used in Discrete Exterior Calculus (DEC) formulations of PDEs. We discovered that while HOT-energy may be a reasonable way to evaluate quality, directly optimizing it, as the community practices, leads to undesirable behavior. In particular, HOT-energy has no barrier to mesh inversion and optimization can collapse elements. Because of these limitations, and the existence of bad local minima, the community relies on preconditioning by Lloyd's iterations as in Centroidal Voronoi Tesselation (CVT). We designed a modified metric that overcomes these shortcomings and is unitless and scale invariant. We believe this new metric will behave better under optimization, while retaining the essential features of bounding the discretization error.

We developed a general framework for constrained local resampling of mesh node positions, where the constraints act analogously to optimization's barriers. Within the local feasible space, we reposition nodes randomly. By locally "satisficing" rather than optimizing quality, nearby nodes retain more positional freedom. Globally, the process is unlikely to get stuck in undesirable local minima. We demonstrated eliminating obtuse angles; nonobtuse triangles have desirable primal-dual properties, such as dual vertices lying inside their primal triangles.

We developed the "VoroCrust" algorithms for generating Voronoi cells whose boundaries match a prescribed domain boundary, useful for polyhedral mesh generation and surface reconstruction. We investigated domains bounded by smooth manifolds, and piece-wise linear complexes. For each we discovered sufficient conditions for the dual Voronoi reconstruction to be provably correct.